DISSERTATION

# MATHEMATICAL MODELING OF URBAN WATER MANAGEMENT STRATEGIES


Submitted by

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WE HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER OUR SUPERVISION BY WYNN R. WALKER ENTITLED MATHEMATICAL MODELING OF URBAN WATER MANAGEMENT STRATEGIES BE ACCEPTED AS FULFILLING IN PART REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY.

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## ABSTRACT OF DISSERTATION

## MATHEMATICAL MODELING OF URBAN WATER MANAGEMENT STRATEGIES

The rapid expansion of many western urban centers has nearly outstripped available stream flow and groundwater resources requiring municipal water departments to initiate exhaustive searches for new water supplies. Among the most feasible alternatives being investigated to date are acquisition and transfer of agricultural rights, interbasin water diversions, and wastewater recycling.

A management level urban water system model has been formulated in which a system analysis format is employed to answer some of the basic questions concerning the optimal combination of these alternative supplies. The model incorporates a non-linear differential optimization algorithm to coordinate urban water supply, distribution, and wastewater management. A test of the model's utility is made in an application to the water management problems of the Denver, Colorado metropolitan area. Denver has utilized both agricultural transfers and transmountain diversions to supplement the natural stream resources of the South Platte River. Although plans are being made to increase the capacity of these sources, increasingly stringent standards on the area's effluents are enhancing the feasibility of reclaiming and recycling a portion of the wastewater. The urban model used in this study indicates the decision

points at which respective strategies are introduced. However, by formulating the model from a planner's viewpoint, the most important results gained from the analysis are the costs of various institutional constraints which may restrict the decision makers' ability to implement optimal policies.

Some of the institutional constraints which have been quantified include the legal interpretation of water right laws, public sentiment towards reuse, consolidation of water supply and wastewater treatment responsibilities, and water quality control philosophies.

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## NOMENCLATURE

| Symbol | Definition | Units |
|---|---|---|
| $A_1, A_2, A_3$ | polynomial regression coefficients | - |
| BOD | biochemical oxygen demand | mg/l |
| b | per capita BOD production | lbs |
| $\underline{C}$ | gradient of the constraints with respect to the decision variables | - |
| $C_k$ | concentration of TDS and BOD at flow control points in models | mg/l |
| $D_k$ | urban demands | mgd |
| $\underline{d}$ | decision variables | - |
| $\underline{f}$ | active constraint set | - |
| $\underline{H}$ | Hession matrix | - |
| $\underline{J}$ | Jacobian matrix | - |
| $P_k$ | constants storing system costs | $ |
| PE | population equivalent | - |
| $Q_k$ | flow rate constants | mgd |
| $\underline{s}$ | state variables | - |
| TDS | total dissolved solids | mg/l |
| $T_k$ | removal efficiencies | % |
| $X_k$ | model flow rate variables | mgd |
| $\underline{x}$ | feasible solutions to optimization problems | - |
| Y | capital construction costs | $ |
| $Y_o$ | operation and maintenance costs | $ |
| y | objective function value and operation and maintenance costs | $ |
| Z | facility capacities | mgd |
| $\phi$ | slack variables | - |
| $\nabla_s$ | gradient with respect to state variables | - |
| $\nabla_d$ | gradient with respect to decision variables | - |
| $\delta$ | constrained derivatives | - |
| $\partial$ | partial derivatives | |

# Chapter 1

# INTRODUCTION

## Purpose

An essential requirement for advancing civilizations has been to increase agricultural production. A few centuries ago a single farmer could barely support his family, but modern agriculturalists are capable of supplying food and fibre for many. The evolution of the agricultural enterprise from the individualistic subsistance farming to the corporate business has also reduced the number of people necessary to satisfy agricultural demands. Consequently, with fewer opportunities in the agricultural industry, people have aggregated in metropolitan environments to work in such tasks as government administration, support services, and manufacturing. A basic shift has thus occurred from rural to urban living.

Regional urbanization has been accompanied by new problems in administering natural resources such as water. First, the demands for water of suitable quality have greatly affected the usefulness of water supplies in several areas. As a result, new sources have been actively sought and the feasibility of employing technological advances to amend marginal supplies has been investigated. Secondly, the concentration of water use in conjunction with the growing demands has created serious water quality degradation by exceeding the natural assimilative capacity of rivers and lakes. And finally, the institutional mechanisms developed to allocate and manage the water resource have not been altered sufficiently to effectively meet the requirements of rapid urbanization.

The purpose of this study is to investigate the feasibility of alternative water management strategies which could be implemented to alleviate the mounting problems of water shortage and water quality deterioration. At this level of interest, the factors especially requiring evaluation are the institutional requirements for accomplishing efficient operation of water use systems. The objective therefore is to model alternative water management strategies and then test the effects of various institutional factors in accomplishing more effective water use.

## Scope

A model of the quantity and quality aspects of an urban water network, with the intention of evaluating management decisions, would incorporate three basic components. These are the water sources (including recycled wastewater), the individual water demands, and the treatment of wastewater. When these parts are combined, a model such as the one illustrated schematically in Figure 1 can be derived. Each of the basic segments of the model is in reality a complex array of physical, economic, social, and political subsystems. However, the detail in which such an urban

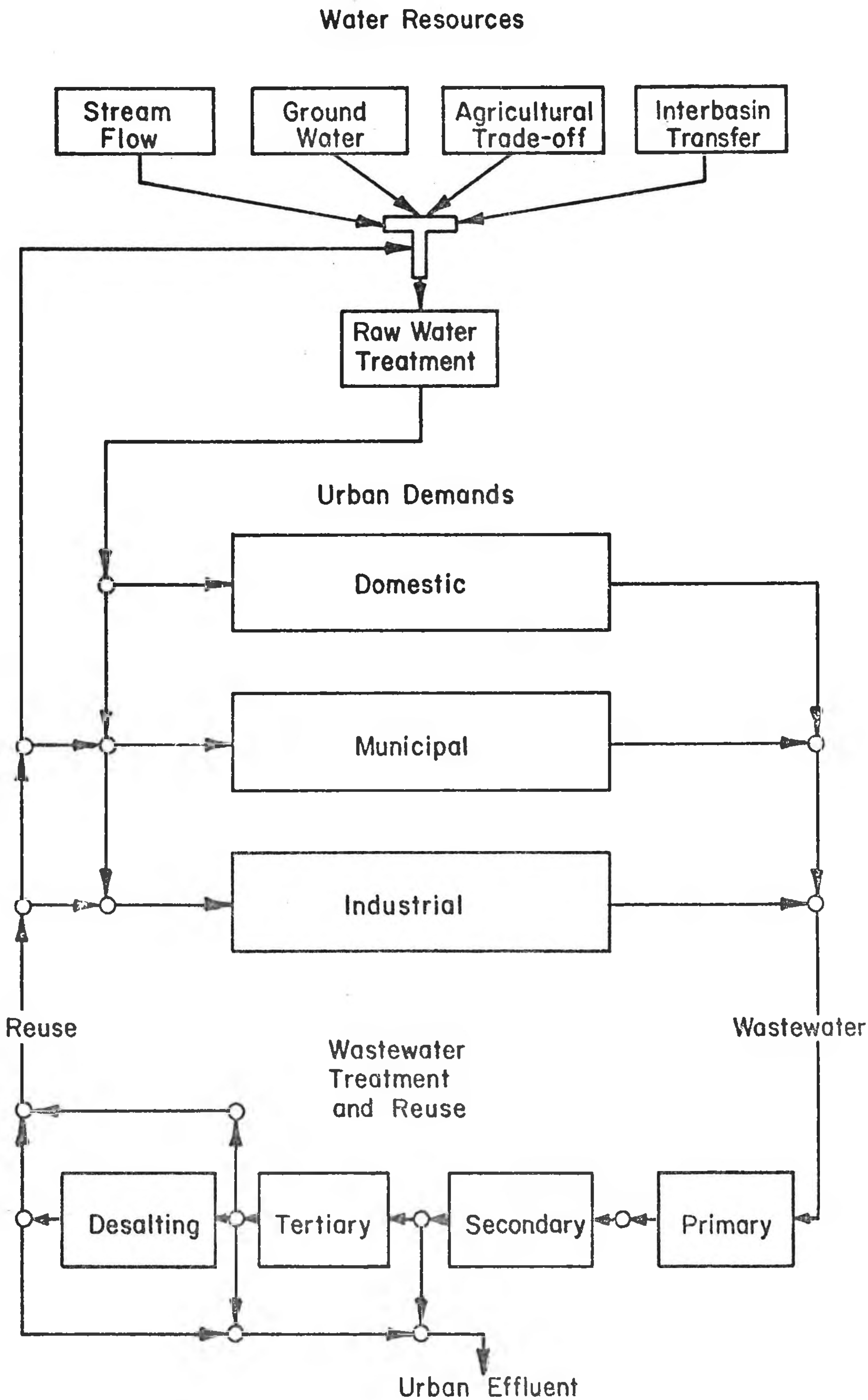


Figure 1. Urban water system model.

model can examine these basic components must be limited to avoid becoming completely entangled in their complexities. Therefore, each of the general parts of the system is examined in macroscopic detail, relying upon future research or technology development to improve the model's reliability and accuracy.

Because the evaluation of management strategies implicitly assumes a systems analysis approach, the initial requirements are a criterion for assessing the feasibility of various alternatives and a method for efficiently selecting the best policy. Consequently, Chapter 2 presents the criterion selected for this study and a general justification for the decision. Then in Chapter 3 a mathematical optimization procedure entitled "Jacobian Differential Algorithm", is derived. The urban water system model is next developed, beginning with the urban wastewater treatment and reclamation model in Chapter 4. Since the wastewater treatment model is necessary for the subsequent use by the urban water supply and distribution model developed in Chapter 5, it has been presented first. These two models are finally coordinated in an application to the conditions of the Denver, Colorado metropolitan area in Chapter 6. A summary, conclusions, and recommendations section is presented in Chapter 7 to finish this work and emphasize the major findings of this analysis.

# Chapter 2

# OPTIMIZATION CRITERION

## Introduction

Alternative measures for meeting the requirements of water management problems in areas of urbanization need to be evaluated for feasibility in the context of both long and short range objectives. In order to facilitate such comparisons necessitates a criterion upon which a common link between alternatives can be developed. This chapter presents some general comment and support from other investigators for the optimization criterion selected for this study.

## Economic Nature of Water Resource Systems

There is probably no other means as commonly used or as widely accepted for evaluating the merits of water resource systems as is economics. While environmental concerns have been mounting and engineering designs have become more sophisticated, the central character in evaluating projects is the economic analysis. Not all of these economic considerations have been made by economists, but those making the studies have of necessity relied upon the discipline to provide new and better techniques for in-investigation.

Although water resources can be classified primarily as public commodities, significant influences on pricing and management are due to water uses in the private market. In most states, water is not legally "owned" by an individual other than the state, but rights can be obtained for the use of water by individuals. However, when the legal interpretation implies that the water is tied to the land and cannot be transferred, then the value of the land is enhanced by its water right. These cases give water a market value obtainable by a right holder even when the resource is administered as public property. As in the case of grazing privileges on public lands, the pricing is usually lower than that obtainable in the private economy. As a consequence, right holders are often reluctant to accept changes which may reduce their water supply.

Reservoirs, diversion works, and distribution systems aid management of water resources which tend to remain fixed in spatial distribution and random in time distribution. These characteristics which would otherwise constrain water supplies to local utilization, allow wider water use between adjoining watersheds and along a river system. However, the diversion of waters from one basin to another, or the transfer of water usage to another location in the river network, creates exteralities which are usually not considered by local planners. Thus, maximum economic efficiencies are only achieved when the economic evaluations assume a regional interpretation.

Finally, the inefficiencies existing in current water use practices can be traced to a large extent to those social, legal, and political institutions responsible for distribution of water among demands. These limitations have not been severe until water resources have become scarce. However, when expansions in urban needs occur, the water resources that could be better utilized in a new use may be tied to an old use without means for making a conversion. As a result, the optimal water management policies which suggest that water be transferred from one use to another, such as transferring agricultural water to municipal uses, have been difficult to date because of the institutional constraints (Hartman and Seastone, 1972). Although such constraints hinder efficient use of water, they have nevertheless become the tools which substitute for the free market economic system.

## Optimizing Criterion

Optimization is generally a maximization or a minimization of concise numerical quantities reflecting the relative importance of the goals and purposes contained in alternative decisions. Of themselves, neither the goals or purposes directly yield the precise quantitative statements required by systems analysis procedures. Therefore, the objectives to be accomplished must first be stated by a quantitative measure from which alternative policies can be mathematically compared (Hall and Dracup, 1970).

Presumably, such a comparison would permit a ranking of these policies as a basis for decision making. The specific measure to facilitate this examination can be defined as the optimizing criterion.

The central problem facing engineers is to link the descriptions of the physical environment via mathematical models with the social and political environment (Thomann, 1972). Probably the most commonly used and widely accepted "indicators" are found among the many economic objective functions. However, considerable controversy exists as to the most realistic of these tools. If all human desires could be priced in an idealized free market monetary exchange, the forces that operated would insure that every individual's marginal costs equalled his marginal gains, thereby insuring maximum economic efficiency. In fact, such a condition would reduce the need for optimization methodologies to aid decision making. In the absence of this ideal situation, goals cannot be quantified with a high degree of accuracy and the optimizing criterion in any case is at best an indicator of the particular alternative.

Among the more adaptable economic indicators are maximization of net benefits, minimum costs, maintaining the economy, and economic development. The use of each depends on the ability to adequately define tangible and intangible direct or indirect costs and benefits. In water resource development and water quality management specifically, the economic incentives for more effective resource utilization are negative in nature (Kneese, 1964). A large part of this problem stems from the fact that water pollution is a cost passed on by the polluter to the downstream user. Consequently, the inability of the existing economic systems to adequately value costs and benefits has resulted in the establishment of water quality standards, however inefficient these may be economically (Hall and Dracup, 1970). The immediate objective of water resource planners is thus to devise and analyze the alternatives for achieving these quality restrictions at minimum cost (Thomann, 1972) and is the criteria chosen for this study.

## Chapter 3

# JACOBIAN DIFFERENTIAL ALGORITHM

### Introduction

The search for an optimizing technique to evaluate the relative merits of an array of alternatives depends largely upon the form of the problem and its constraints. While the allegorical Chinese maxim cited by Wilde and Beightler (1967) stating, "There are many paths to the top of the mountain, but the view there is always the same," is also true in this case; not every method can be applied with the same ease. Each optimization scheme has its unique properties making it adaptable to specific problems, although many techniques when sufficiently understood can be modified to extend their applicability. Successful modifications of this nature are prevalent in current engineering practice but requires some experience in using these methods.

Most conditions encountered in the field of water resources, urban water systems specifically, involve mathematical formulations which are non-linear in both the objective function and the constraints. Furthermore, the constraining functions may be mixtures of linear and non-linear equalities and inequalities. Without simplifying these problems or radically changing existing optimization techniques, it is possible to derive solutions based upon what Wilde and Beightler (1967) describe as the "differential appraoch."

Most techniques for selecting the optimal policy do so by successively improving a previous estimate until no betterment is possible. These may be classified as direct or indirect methods depending on whether they start at a feasible point and stepwise move toward the optimum or solve a set of equations which contain the optimum as a root. In a majority of cases, the differential approach can be used to describe the method. Thus, it is possible to understand a wide variety of procedures by knowing one basic mathematical approach.

Numerous applications of one form or another of the basic differential approach have been made in the field of engineering (Monarchi, 1972). Because of the considerable difficulty in programming "generality," nearly all of these applications have been somewhat specialized toward the specific geometry of the problem. The research project responsible for this development necessitates two entirely different optimization analyses. Consequently, to avoid developing two models, it was decided to attempt to program a general differential algorithm. A class entitled "Foundations of Engineering Optimization" taught by Dr. H. J. Morel-Seytoux, Professor of Civil Engineering at Colorado State University provided the theoretical basis for the model. This writer, a student in the class, coded the algorithm for use on the digital computer facilities at the University.

The optimizing technique is called in this writing the "Jacobian Differential Algorithm." Theoretically, it is a generalized eliminating procedure which is computationally feasible under a wide variety of conditions. The characteristics of convexity are assumed and since the maximization problem is simply the negative of a minimization one, the succeeding discussion will be limited to the latter case. As in all direct minimizing procedures, the algorithm involves four steps:

1. Evaluate a first feasible solution, $\underline{x}^o$ which satisfies the problem constraints. The underbar indicates vector notation and the superscript $^o$ is used to describe the "old" or initial points.
2. Determine the direction in which to move such that the objective function, y, is decreased the most rapidly. This requires a move from $\underline{x}^o$ to the new point, $\underline{x}^\nu$ in which the superscript $^\nu$ represents the new point notation.
3. Find the distance that can be moved without violating any of the problem constraints.
4. Stop when the optimum is reached.

While the procedure yields the requirements for steps 2 and 3, the user is left with providing the first feasible solution, step 1. This may seem to be a drawback for the problem, but in real situations a feasible solution already exists as a current policy. Step 4 is accomplished by an examination of what are now referred to as the "Kuhn-Tucker conditions." These criteria do not indicate whether the procedure has reached a local or global optimum; consequently, it is necessary to derive a means for checking. This is not usually a difficult process.

## Theoretical Development

Consider the problem in which the minimum value of the objective function is sought subject to a set of constraining functions. Writing this problem mathematically,

$$\min_{x} \{y = y(\underline{x})\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

subject to,

$$\underline{f}(\underline{x}) \geq \underline{0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

where the notation $y(\underline{x})$ denotes "as a function of the vector x." The number of x variables is defined as N and the number of constraints as K. The method of analysis depends largely upon the structure of the constraints. When all the constraints are inequalities and "loose" or "inactive" (strictly >) at the initial feasible point $\underline{x}^o$, the problem is "unconstrained." In the other case when either some of these functions are strict equalities or when some of the inequalities are "tight" or "active," the problem is referred to as "constrained." Although both of the conditions may occur in the solution of a problem, they require somewhat different approaches as the algorithm progresses toward the optimum.

Elimination Procedure

The elimination nature of the technique is derived from the fact that it is at least conceptually possible to employ only the currently active constraints to eliminate some of the x's from the problem, making it temporarily unconstrained. To begin, define the number of active constraints as T and reorder the constraint set so that the first T are the active constraints with index t = 1, 2, . . ., T. Further, introduce "slack" variables to the active constraints so they take the form,

$$\underline{f}(\underline{x}) - \underline{\phi} = \underline{0} \qquad \ldots \qquad (3)$$

and become strict equalities, where $\underline{\phi}$ is the vector of slack variables. Later in the development, slack variables will also be added to the inactive constraints. The purpose of this transformation is that by continual observation of the slack values, the distinction between active and inactive functions can be determined, since active slack variables are equal to zero and inactive slacks are always greater than zero. The problem now contains N original variables plus T slack variables which are related by T active constraints. If the constraints are linear, T of the variables can be eliminated from the objective function by the constraint expressions, making the problem unconstrained. However, in the general situation, the constraints are nonlinear, and it is not directly possible to substitute for the dependent variables. It is necessary in the general case to first linearize the functions by taking the first partial derivatives with respect to the x variables. Even though the non-linearity may still exist due to the nature of the terms in the constraints, if it is assumed that the changes toward the optimum point are sufficiently small, then only a small deviation is introduced. The elimination procedure takes place by partitioning the variable set into "states" and "decisions." The state variables are the selected variables which are to be eliminated by the T active constraints. The decision variables are the remaining independent variables which will be employed to seek the minimum value of the objective function. The criteria for the partition include two aspects:

1. All slack variables are taken as decisions unless no other x-variable is available to be a state variable. Since all $\phi_t$ are identically equal to zero, when the algorithm moves from the old point $\underline{x}^o$ to the new one $\underline{x}^\nu$ in its search for the minimum, there is a 50 percent chance that the $\phi_t$ will become negative. This is a violation of the problem constraints.
2. Since the same basic reasoning applies to the x-variables, the largest absolute valued variables are best suited to be state variables.

In the computer code of the algorithm, the selection of states and decisions is much more complex, but to describe all the partitioning difficulties at this point would be confusing.

After partitioning the x-vector into state and decision variables, the variables can be relabeled s for states and d for decisions. Equation 1 at the initial point $\underline{x}^{o}$ can then be written,

$$\min_{\underline{d}} \quad y = y(s_1, s_2, \ldots, s_T, d_1, d_2, \ldots, d_D) \quad . \quad . \quad (4)$$

in which D is the number of decision variables and equals (N - T). In addition, the constraints listed in Equation 3 can be rewritten as:

$$\underline{f}(\underline{s},\underline{d}) - \underline{\phi} = \underline{0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The next step is to employ the chain rule of calculating the total differential of y. In vector notation,

$$\partial y = (\nabla_s y)\partial\underline{s} + (\nabla_d y)\partial\underline{d} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where the symbol $\partial y$ is used to denote the total differential rather than the standard notation of dy. This modification is made so that the d can be reserved to denote the decision variables.

The derivatives of the constraining functions can also be written in vector form,

$$(\nabla_s\underline{f})\partial\underline{s} + (\nabla_d\underline{f})\partial\underline{d} - \partial\underline{\phi} = \underline{0} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where the gradient, $(\nabla_s\underline{f})$, is called the Jacobian Matrix, $\underline{J}$, and the matrix $(\nabla_d\underline{f})$ can be relabeled as $\underline{C}$. Employing these variables in Equation 7 and rearranging terms:

$$\underline{J}\partial\underline{s} = -\underline{C}\partial\underline{d} + \partial\underline{\phi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

If the Jacobian matrix is always taken non-singular, the vector $\partial\underline{s}$ can be solved for.

$$\partial \underline{s} = -\underline{J}^{-1}\underline{C}\partial\underline{d} + \underline{J}^{-1}\partial\underline{\phi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

The elimination of the states is now possible by substitution of Equation 9 into Equation 6. After rearranging terms, the final unconstrained equation is developed.

$$\partial y = \left[\nabla_d y - (\nabla_s y)\underline{J}^{-1}\underline{C}\right] \underline{\partial d} + (\nabla_s y)\underline{J}^{-1}\partial\underline{\phi} \qquad . \quad . \qquad (10)$$

Kuhn-Tucker Conditions

At this point, the key parameters in the Jacobian Differential Algorithm can be introduced. By definition of the total differential, another expression can be written in terms of the variables indicated in Equation 10. If the elimination of the state differentials was accomplished then the total differential of y would be written,

$$\partial y = \frac{\delta y}{\delta \underline{d}}\, \partial\underline{d} + \frac{\delta y}{\delta \underline{\phi}}\, \partial\underline{\phi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (11)$$

in which $\delta y/\delta \underline{d}$ and $\delta y/\delta \underline{\phi}$ are called "constrained derivatives." The deviation in notation is made to distinguish the $\partial y/\partial \underline{x}$, which is a partial derivative viewing all variables as independent, from $\delta y/\delta \underline{d}$ which is a partial derivative considering T of the variables as functions of the remaining N variables. By comparing Equations 10 and 11 it can be seen that,

$$\frac{\delta y}{\delta \underline{d}} = \nabla_d y - (\nabla_s y)\underline{J}^{-1}\underline{C} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (12)$$

and,

$$\frac{\delta y}{\delta \underline{\phi}} = (\nabla_s y)\underline{J}^{-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (13)$$

The solution of Equations 12 and 13 when equated to zero yield a stationarity point when the decision variables are free, or in other words, allowed to assume any positive or negative value. In most instances, decision variables are not free, but subject to non-negativity conditions. Stationarity points may be local or global minimums, maximums, or inflection points. The evaluation of stationarity points in these cases will depend on criteria reported by Kuhn and Tucker (1951) which provide necessary and sufficient conditions for a minimum. In the problem solution at the feasible point under examination, a minimum exists if the following conditions are met:

1. Necessary conditions prerequisite for a minimum must consist of the following:

$$\frac{\delta y}{\delta d_j} \geq 0,\ d_j \geq 0,\ \text{and}\ \frac{\delta y}{\delta d_j} d_j = 0 \quad j = 1,2,\ldots,D \qquad (14)$$

and

$$\frac{\delta y}{\delta \phi_t} \geq 0,\ \phi_t \geq 0,\ \text{and}\ \frac{\delta y}{\delta \phi_t} \phi_t = 0,\ t = 1,2,\ldots,T \qquad (15)$$

2. If Equations 14 and 15 are satisfied, then sufficient conditions for a minimum are:

$$\frac{\delta y}{\delta d_j} > 0 \quad j = 1,2,\ldots,D \quad . \quad . \quad . \quad . \qquad (16)$$

and

$$\frac{\delta d}{\delta \phi_t} > 0 \quad t = 1,2,\ldots,T \quad . \quad . \quad . \quad . \qquad (17)$$

The minimum has been reached when both the necessary and sufficient conditions have been satisfied. However, if for example, $\delta y/\delta d_j$ equals zero and $d_j \geq 0$, the tests are inconclusive since the sufficient conditions have not been met. In this case, it is necessary to take the second derivatives of the objective function with respect to the x-vector. This analysis yields a square matrix of second order partial derivatives called the Hessian matrix written mathematically as:

$$\underline{H} = \nabla_x^2 \, y \quad . \quad . \quad . \quad . \quad . \quad . \qquad (18)$$

In order for the stationarity point to be a minimum (local or global) the value of the Hessian matrix must be positive-definite, and since the properties of positive-definite matrices can be found in most texts on linear algebra, no further description will be given here.

## Evaluation of Optimal Direction

In addition to the description of the fundamental elimination technique of this optimizing technique, the preceding sections also provided the definition of the constrained derivatives of the objective function in terms of the decision and slack variables. Furthermore, criteria were given with which these parameters can also be evaluated to see when the minimum is achieved. In this section, these same derivatives will be used to determine the direction a particular decision variable, $d_p$ or $\phi_p$, must be "moved" in order to create the maximum reduction in the value of the objective function during each iterative step.

Among the non-linear programming techniques for optimization several essentially alter all of the decision variables at each iteration. In the Jacobian Differential Algorithm, one decision variable ($d_p$ or $\phi_p$) is selected from among the set which when moved will result in the most progress toward the minimum. If an individual term from Equation 11 is written in discrete element form, the new value of the decision variable (or slack variable) can be determined,

$$y^{\nu} - y^{o} = \left(\frac{\delta y}{\delta d_i}\right)^{o} \left(d_i^{\nu} - d_i^{o}\right) \quad . \quad . \quad . \quad . \qquad (19)$$

or,

$$y^{\nu} - y^{o} = \left(\frac{\delta y}{\delta \phi_t}\right)^{o} \phi_t^{\nu} \quad . \quad . \quad . \quad . \quad . \qquad (20)$$

where the reader is reminded that the superscripts $^{o}$ and $^{\nu}$ refer to the functional evaluations made at the old and new feasible solutions. It may also be worth mentioning that $\phi_t$ can only be increased whereas $d_i$ can be also decreased (assuming the non-negativity constraints are not violated). As a result, the increase in a slack variable is in reality a loosening of an active constraint.

The choice of the decision variable or the slack variable to be modified is primarily made on the basis of largest absolute value among the respective constrained derivatives. Three general categories are examined. To begin with, the largest positive valued derivative with which the associated decision variable is greater than zero is determined and the Kuhn-Tucker Conditions are checked according to the previous section. Mathematically, this first alternative can be written,

$$\text{find:} \quad \max_i \left[ \frac{\delta y}{\delta d_i} > 0 \quad \middle| d_i > 0,\ i = 1,2,. . .,D \right] \qquad (21)$$

where the notation $|d_i > 0$ means "subject to the value of $d_i$ being positive."

The second alternative selection for the step direction is in the negative constrained derivatives. In this case, the specific decision variable will be increased and unless an upper bound on the variable is imposed, no examination of the decision need be made. Symbolically then,

$$\text{find:} \quad \min_i \left[ \frac{\delta y}{\delta d_i} < 0,\ i = 1,2,. . ., D \right] . \quad . \quad . \qquad (22)$$

Finally, the largest reduction in the objective function may be facilitated by loosening a particular active constraint. Unless the constrained derivative of y with respect to the slack variable is negative, the Kuhn-Tucker Conditions are satisfied. Therefore, this solution can be expressed as:

$$\text{find:} \quad \min_t \left[ \frac{\delta y}{\delta \phi_t} > 0,\ t = 1,2,. . ., T \right] \quad . \quad . \qquad (23)$$

Once these maximum and minimums have been selected, the next item is to compare them with each other and select the largest absolute valued one. After having made the choice, the index on the specified decision or slack variable is now denoted by a "p", and these variables now become $d_p$ or $\phi_p$ depending on the decision among alternatives.

## Determining the Step Size

The analysis in the previous section paved the way to compute the direction in which the decision and slack variables are to be moved for a maximum decrease in the objective function. This section is presented to find how much the procedure can move in the appropriate direction without violating the constraints. In order to accomplish this, four new constrained derivatives must be developed. To begin, it is useful to rewrite Equations 6 and 7 as a complete differential system. In addition, the slack variables ($\phi^+$) may be added to the inactive constraints ($f^+$) and included in the differential system. The complete three-part system has been included in Figure 2.

Because the particular decision variable or slack variable to be modified has been selected, the remaining decisions and slacks will remain constant and can therefore be temporarily ignored. The next computation necessary is to determine which of the boundaries of the problem are approached first. If the non-negativity constraints on the variables are in effect, one consideration is how far a decision or slack variable can be moved without forcing a state variable to become negative. In order to accomplish this, the constrained derivatives of each state variable with respect to the particular decision or slack variable are computed. The representation of these values is computed from the formulas shown in Figure 3 in which the use of Cramer's rule was applied to the system in Figure 2.

$$-\partial y + \frac{\partial y}{\partial s_1}\partial s_1 + \frac{\partial y}{\partial s_2}\partial s_2 + \dots + \frac{\partial y}{\partial s_T}\partial s_T = -\frac{\partial y}{\partial d_1}\partial d_1 - \frac{\partial y}{\partial d_2}\partial d_2 - \dots - \frac{\partial y}{\partial d_D}\partial d_D$$

$$\begin{aligned}
&\frac{\partial f_1}{\partial s_1}\partial s_1 + \frac{\partial f_1}{\partial s_2}\partial s_2 + \dots + \frac{\partial f_1}{\partial s_T}\partial s_T = -\frac{\partial f_1}{\partial d_1}\partial d_1 - \frac{\partial f_1}{\partial d_2}\partial d_2 - \dots - \frac{\partial f_1}{\partial d_D}\partial d_D + \partial\phi_1 \\
&\vdots \\
&\frac{\partial f_t}{\partial s_1}\partial s_1 + \frac{\partial f_t}{\partial s_2}\partial s_2 + \dots + \frac{\partial f_t}{\partial s_T}\partial s_T = -\frac{\partial f_t}{\partial d_1}\partial d_1 - \frac{\partial f_t}{\partial d_2}\partial d_2 - \dots - \frac{\partial f_t}{\partial d_D}\partial d_D + \partial\phi_t \\
&\vdots \\
&\frac{\partial f_T}{\partial s_1}\partial s_1 + \frac{\partial f_T}{\partial s_2}\partial s_2 + \dots + \frac{\partial f_T}{\partial s_T}\partial s_T = -\frac{\partial f_T}{\partial d_1}\partial d_1 - \frac{\partial f_T}{\partial d_2}\partial d_2 - \dots - \frac{\partial f_T}{\partial d_D}\partial d_D + \partial\phi_T
\end{aligned}$$

$$\begin{aligned}
&-\partial\phi_1^+ + \frac{\partial f_1^+}{\partial s_1}\partial s_1 + \frac{\partial f_1^+}{\partial s_2}\partial s_2 + \dots + \frac{\partial f_1^+}{\partial s_T}\partial s_T = -\frac{\partial f_1^+}{\partial d_1}\partial d_1 - \frac{\partial f_1^+}{\partial d_2}\partial d_2 - \dots - \frac{\partial f_1^+}{\partial d_D}\partial d_D \\
&\vdots \\
&-\partial\phi_\ell^+ + \frac{\partial f_\ell^+}{\partial s_1}\partial s_1 + \frac{\partial f_\ell^+}{\partial s_2}\partial s_2 + \dots + \frac{\partial f_\ell^+}{\partial s_T}\partial s_T = -\frac{\partial f_\ell^+}{\partial d_1}\partial d_1 - \frac{\partial f_\ell^+}{\partial d_2}\partial d_2 - \dots - \frac{\partial f_\ell^+}{\partial d_D}\partial d_D \\
&\vdots \\
&-\partial\phi_L^+ + \frac{\partial f_L^+}{\partial s_1}\partial s_1 + \frac{\partial f_L^+}{\partial s_2}\partial s_2 + \dots + \frac{\partial f_L^+}{\partial s_T}\partial s_T = -\frac{\partial f_L^+}{\partial d_1}\partial d_1 - \frac{\partial f_L^+}{\partial d_2}\partial d_2 - \dots - \frac{\partial f_L^+}{\partial d_D}\partial d_D
\end{aligned}$$

Figure 2. Differential system expressing the linearized objective function, active constraints, and inactive constraints.

$$\frac{\delta s_i}{\delta d_p} = - \frac{\begin{vmatrix} \frac{\partial f_1}{\partial s_1} & \cdots & \frac{\partial f_1}{\partial s_{i-1}} & \frac{\partial f_1}{\partial d_p} & \frac{\partial f_1}{\partial s_{i+1}} & \cdots & \frac{\partial f_1}{\partial s_T} \\ \vdots & & & & & & \\ \frac{\partial f_t}{\partial s_1} & \cdots & \frac{\partial f_t}{\partial s_{i-1}} & \frac{\partial f_t}{\partial d_p} & \frac{\partial f_t}{\partial s_{i+1}} & \cdots & \frac{\partial f_t}{\partial s_T} \\ \vdots & & & & & & \\ \frac{\partial f_T}{\partial s_1} & \cdots & \frac{\partial f_T}{\partial s_{i-1}} & \frac{\partial f_T}{\partial d_p} & \frac{\partial f_T}{\partial s_{i+1}} & \cdots & \frac{\partial f_T}{\partial s_T} \end{vmatrix}}{|\underline{J}|}$$

$$\frac{\delta s_i}{\delta \phi_p} = (-1)^{t+i} \frac{\begin{vmatrix} \frac{\partial f_1}{\partial s_1} & \cdots & \frac{\partial f_1}{\partial s_{i-1}} & \frac{\partial f_1}{\partial s_{i+1}} & \cdots & \frac{\partial f_1}{\partial s_T} \\ \vdots & & & & & \\ \frac{\partial f_{t-1}}{\partial s_1} & \cdots & \frac{\partial f_{t-1}}{\partial s_{i-1}} & \frac{\partial f_{t-1}}{\partial s_{i+1}} & \cdots & \frac{\partial f_{t-1}}{\partial s_T} \\ \frac{\partial f_{t+1}}{\partial s_1} & \cdots & \frac{\partial f_{t+1}}{\partial s_{i-1}} & \frac{\partial f_{t+1}}{\partial s_{i+1}} & \cdots & \frac{\partial f_{t+1}}{\partial s_T} \\ \vdots & & & & & \\ \frac{\partial f_T}{\partial s_1} & \cdots & \frac{\partial f_T}{\partial s_{i-1}} & \frac{\partial f_T}{\partial s_{i+1}} & \cdots & \frac{\partial f_T}{\partial s_T} \end{vmatrix}}{|\underline{J}|}$$

Figure 3. Algebraic formulas for computing the state variable constrained derivatives with respect to the particular decision or slack variables.

From these values, the maximum move may be computed. Writing the appropriate relationships in discrete form,

$$(s_i^{\nu} - s_i^{o}) = \left(\frac{\delta s_i}{\delta d_p}\right)^{o} \left(d_p^{\nu} - d_p^{o}\right) \quad . \quad . \quad . \quad . \qquad (24)$$

or for the slack variables:

$$(s_i^{\nu} - s_i^{o}) = \left(\frac{\delta s_i}{\delta \phi_p}\right)^{o} \quad \phi_p^{\nu} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (25)$$

Three cases exist in which a state variable can be driven to zero, namely a decrease in $d_p$, an increase in $d_p$, and an increase (or loosening) in $\phi_p$. Since a search is necessary among the state variables to see which specific state goes to zero first, Equations 24 and 25 can be incorporated:

Case 1. Decreasing $d_p$

$$d_p^{\nu} = \max_i \left[ d_p^{o} - \frac{s_i^{o}}{\left(\frac{\delta s_i}{\delta d_p}\right)^{o}} \;\middle|\; \frac{\delta s_i}{\delta d_p} > 0 \right] \quad . \quad . \qquad (26)$$

Case 2. Increasing $d_p$

$$d_p^{\nu} = \min_i \left[ d_p^{o} - \frac{s_i^{o}}{\left(\frac{\delta s_i}{\delta d_p}\right)^{o}} \;\middle|\; \frac{\delta s_i}{\delta d_p} < 0 \right] \quad . \quad . \qquad (27)$$

Case 3. Increasing $\phi_p$.

$$\phi_p^{\nu} = \min_i \left[ - \frac{s_i^{o}}{\left(\frac{\delta s_i}{\delta \phi_p}\right)^{o}} \;\middle|\; \frac{\delta s_i}{\delta \phi_p} < 0 \right] \quad . \quad . \qquad (28)$$

The next possible limitation on the change in the decision or slack variables is the forcing of a previously inactive constrant into an active role in the problem. In order to facilitate this analysis, the constrained derivatives of the loose slack variables with respect to the decision and slack variables is computed. A formula for these computations is given in Figure 4. Again, three conditions must be considered:

Case 1. Decreasing $d_p$

$$d_p^{\nu} = \max_{\ell} \left[ d_p^{o} - \frac{(\phi_\ell^+)^o}{\left(\frac{\delta f_\ell^+}{\delta d_p}\right)^o} \;\middle|\; \frac{\delta f_\ell^+}{\delta d_p} > 0 \right] \quad . \quad . \quad (29)$$

Case 2. Increasing $d_p$

$$d_p^{\nu} = \min_{\ell} \left[ d_p^{o} - \frac{(\phi_\ell^+)^o}{\left(\frac{\delta f_\ell^+}{\delta d_p}\right)^o} \;\middle|\; \frac{\delta f_\ell^+}{\delta d_p} < 0 \right] \quad . \quad . \quad (30)$$

Case 3. Increasing $\phi_p$

$$\phi_p^{\nu} = \min_{\ell} \left[ - \frac{(\phi_\ell^+)^o}{\left(\frac{\delta f_\ell^+}{\delta \phi_p}\right)^o} \;\middle|\; \frac{\delta f_\ell^+}{\delta \phi_p} < 0 \right] \quad . \quad . \quad (31)$$

A final limitation which should be noted is when a decrease in $d_p$ is to be made and neither condition above is violated before non-negativity is encountered. In such a case, the maximum decrease would be $- d_p$ assuming the non-negativity conditions hold. Once this and the other values of $d_p$ and $\phi_p$ have been made, the most limiting case is evaluated as the proper change in $d_p$ or $\phi_p$, whichever the case may be.

$$\frac{\delta f_\ell^+}{\delta d_p} = \frac{\begin{vmatrix} \frac{\partial f_\ell^+}{\partial d_p} & \frac{\partial f_\ell^+}{\partial s_1} & \cdots & \frac{\partial f_\ell^+}{\partial s_T} \\ \frac{\partial f_1}{\partial d_p} & \frac{\partial f_1}{\partial s_1} & \cdots & \frac{\partial f_1}{\partial s_T} \\ \vdots & & & \\ \frac{\partial f_t}{\partial d_p} & \frac{\partial f_t}{\partial s_1} & \cdots & \frac{\partial f_t}{\partial s_T} \\ \vdots & & & \\ \frac{\partial f_T}{\partial d_p} & \frac{\partial f_T}{\partial s_1} & \cdots & \frac{\partial f_T}{\partial s_T} \end{vmatrix}}{|\underline{J}|}$$

$$\frac{\partial f_\ell^+}{\partial \phi_p} = (-1)^{p+1} \frac{\begin{vmatrix} \frac{\partial f_\ell^+}{\partial s_1} & \cdots\cdots\cdots & \frac{\partial f_\ell^+}{\partial s_T} \\ \frac{\partial f_1}{\partial s_1} & \cdots\cdots\cdots & \frac{\partial f_1}{\partial s_T} \\ \vdots & & \\ \frac{\partial f_{p-1}}{\partial s_1} & \cdots\cdots\cdots & \frac{\partial f_{p-1}}{\partial s_T} \\ \frac{\partial f_{p+1}}{\partial s_1} & \cdots\cdots\cdots & \frac{\partial f_{p+1}}{\partial s_T} \\ \vdots & & \\ \frac{\partial f_T}{\partial s_1} & \cdots\cdots\cdots & \frac{\partial f_T}{\partial s_T} \end{vmatrix}}{|\underline{J}|}$$

Figure 4. Formulas for calculation of the loose slack variable constrained derivatives with respect to the particular decision or slack variables.

Before this section is concluded, a few notes should be made. The first of these is that the number of state variables depends only on the number of active constraints. If by varying a slack or decision variable, a state is driven to zero, a decision variable must be selected to trade positions with the state because of the rick of zero valued state variables. The second point to make is that when a loose constraint is tightened, a new state variable must be selected from the rest of the decision variables. The exception to this is when a loose constraint is tightened by loosening a currently active constraint. In any event, there are so many functions and variables to keep track of, and so many possible alternatives to consider, that the most difficult aspect of this algorithm is the "bookkeeping" that is necessary. This will be demonstrated in the discussion of the computer code.

## The Computer Code

Although the theory encompassing this optimization technique is a very powerful one, the computer code of the method has certain inherent limitations. This is not a fault of this particular program, but rather a characteristic of nearly all programs with any degree of sophistication. The utility of any optimum seeking procedure in engineering applications is largely dependent on the economy of use and its generality. It is primarily the latter aspect that limits the subsequent use by an individual unfamilar with the mechanics of the programs' operation. Very few large computer programs are general enough to be used with little or no knowledge of their structure and weak points. The computer code developed in this section is not among these very few, but a great deal of time and effort has been spent in maximizing the generality of the program.

One of the most efficient uses of coding technology is to provide the means whereby segments of programs can be easily modified and used successively for other purposes. In order to facilitate future use of this program, each functional element in the procedure has been identified in a subroutine format. This type of program structure has several important advantages including the ease in which the program can be debugged. In addition, whatever modifications become desirable can be made within the framework of the subroutine without detailed consideration to the remainder of the program. Another advantageous characteristic of the program is that most of the variables are placed in a common storage, thereby making their values accessable from throughout the program.

The Jacobian Differential Algorithm consists of 25 subroutines which have been defined in Table 1. The entire system can be subdivided into seven groups according to their role in the optimizing technique:

1. Problem definition is accomplished in

Table 1. Definition of subroutine functions.

| Subroutine | Function |
|---|---|
| ANSOUT | Output of the optimal solution |
| ARRAY | Determination of initial variable partition |
| CONDER | Updates values contained in program storage arrays |
| DATAOUT | Output of input data and control variables |
| DECDJ | Decreases the value of a decision variable |
| DFDX | Derivatives of the constraints, $\partial f/\partial x$ |
| DIFALGO | Coordination of the complete algorithm |
| DYDX | Derivatives of the objective function, $\partial y/\partial x$ |
| ENDCHEK | Checks problem to insure the search remains in a feasible region |
| FKOFX | Constraints |
| GAUSS | Gaussian elimination procedure for solving system of linear equations |
| INCDJ | Increases the value of a decision variable |
| INCFT | Loosens a previously active constraint |
| JACOBI | Computation of the determinant of the Jacobian matrix |
| JORK | Selection of the decision or slack variable resulting in the most decrease in the value of the objective function |
| KODFLDD | Constrained derivative, $\delta\phi_\ell^+/\delta d_p$ |
| KODFLDF | Constrained derivative, $\delta\phi_\ell^+/\delta\phi_p$ |
| KODSDD | Constrained derivative, $\delta s_i/\delta d_p$ |
| KODSDF | Constrained derivative, $\delta s_i/\delta\phi_p$ |
| KODYDD | Constrained derivative, $\delta y/\delta d_j$ |
| KODYDF | Constrained derivative, $\delta y/\delta\phi_j$ |
| KUNTUK | Checks Kuhn-Tucker conditions for a minimum |
| MPREG | Utility routine for polynomial regressions |
| NEWTSIM | Newton-Raphson method for solving systems of non-linear equations |
| YOFX | Computes the value of the objective function |

subroutines YOFX, FKOFX, DYDX, and DFDX.

2. Input-Output is provided by the subroutines DATAOUT and ANSOUT.
3. The coordination of the entire program procedure is handled in subroutine DIFALGO.
4. Organization functions in the program are completed in subroutines REORGA and ARRAY.
5. Special computational subroutines include JORK, JACOB1, ENDCHEK, CONDER, KUNTUK, NEWTSIM, and GAUSS.
6. The principal parts of the program are encompassed in subroutines DECDJ, INCDJ, and INCFT which accomplish the step-by-step movement toward the optimum.
7. The calculation of the constrained derivatives is done in the subroutines, KODYDD, KODYDF, KODSDD, KODFLDD, KODSDF, and KODFLDF.

Although each of these subroutines have certain independent functions, it is probably only worthwhile to describe a select few so the reader can observe the basic operation of the program. The most useful illustrations of the program's operation are best given by a detailed examination of the subroutines DIFALGO, REORGA, NEWTSIM, and DECDJ.

## Subroutine DIFALGO

The basic procedure of this differential algorithm is contained in the subroutine DIFALGO where the minimization technique is coordinated. Aside from whatever peripheral program that might be using the algorithm for some phase of its operations, the primary control in the program itself is in the subroutine DIFALGO. A detailed flow chart of this subroutine is illustrated in Figure 5.

After entering DIFALGO, the first step is the initialization of certain internal control variables, as well as an array variable necessary in later computations. Then, an interative loop is entered in which a prescribed number of steps toward the minimum will be taken, or until the minimum is reached within satisfactory tolerances. The value of the objective function and the constraint slack variables are next calculated by calling subroutines YOFX and FKOFX. Information from the latter is subsequently used to determine both the number of active and inactive constraints so the number of state variables can be determined. Control is then shifted to subroutine ARRAY for the first initial partition between state and decision variables, and active and inactive constraints. Calling the subroutines DYDX and DFDX provides the values of the objective function and constraint derivatives which are next used in the subroutine REORGA, which reorganizes this data according to the variable partition accomplished in subroutine ARRAY and then checks the determinant of the Jacobian matrix, $\underline{J}$ , to insure non-singularity. Then, DIFALGO calls the subroutines KODYDD and KODYDF, which provide the values of the constrained derivatives of the objective function with respect

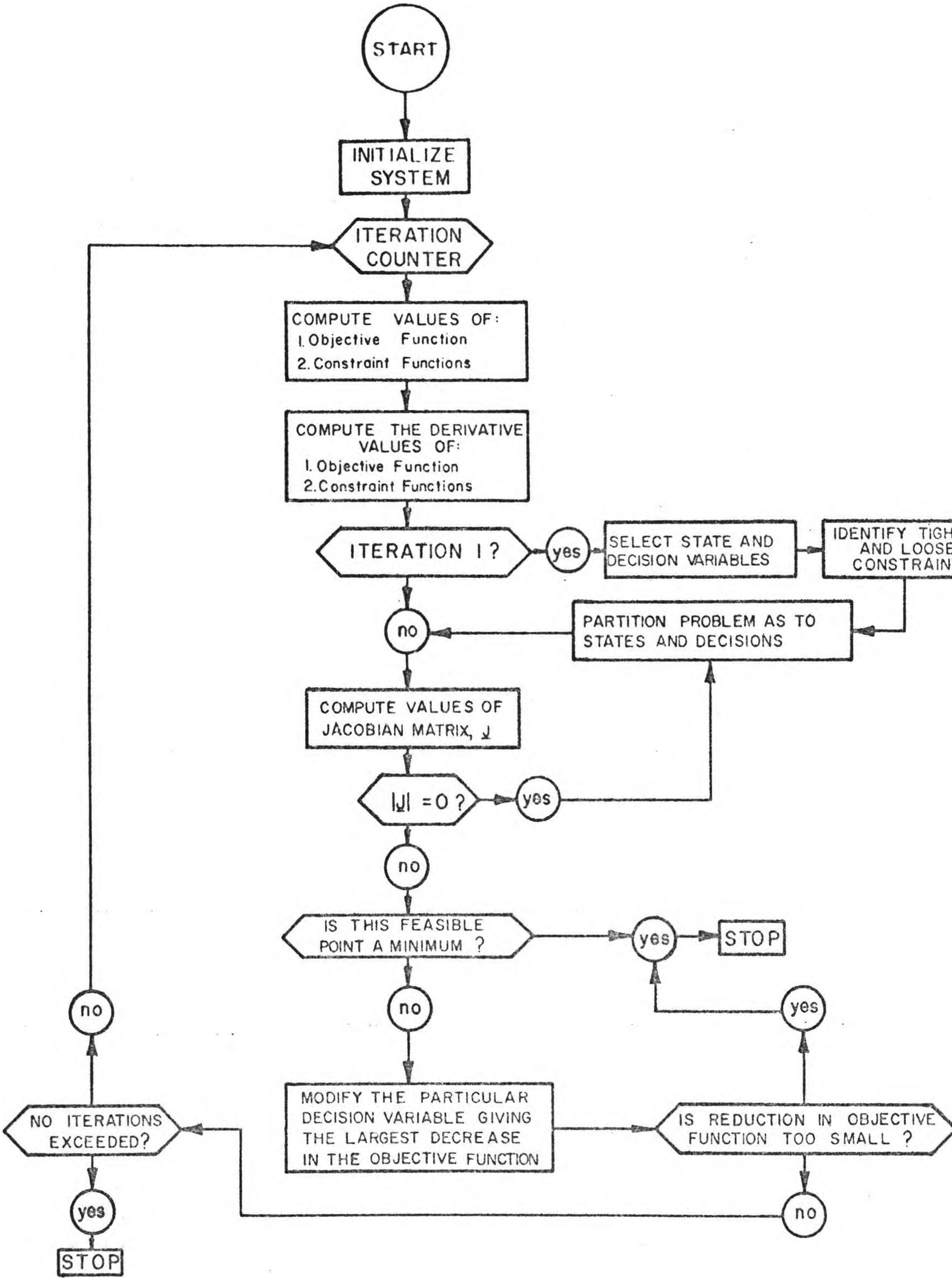


Figure 5. Illustrative flow chart of the subroutine DIFALGO.

to the decision and slack variables. These values are then used in subroutine JORK to determine which decision or slack variable is to be modified. The Kuhn-Tucker conditions are next checked; if they are satisfied, the procedure succeeded. Control is then passed to the appropriate change function (decrease $d_p$, DECDJ, increase $d_p$, INCDJ, or loosen a tight constraint, INCFT) where the step toward the optimum is taken and all problem boundaries are checked for violations. Finally, the program returns to the next iteration.

Subroutine REORGA

REORGA is essentially a bookkeeping and filing subroutine necessary to managing the continual changes that occur in the immediate structure of the problem. It is called not only from DIFALGO, but also from each step in the subroutines responsible for changing the decision and slack variables. A detailed flow chart of this subroutine is presented in Figure 6.

Upon the transfer of control to REORGA, the subroutine's first task is to relabel the derivatives of the objective function, active constraints, and inactive constraints with respect to the x variable defined in the problem formulation into derivatives of these parameters with respect to state, decision, and slack variables. Once this function has been completed, the subroutine JACOB1 is called where the Jacobian matrix is defined and its

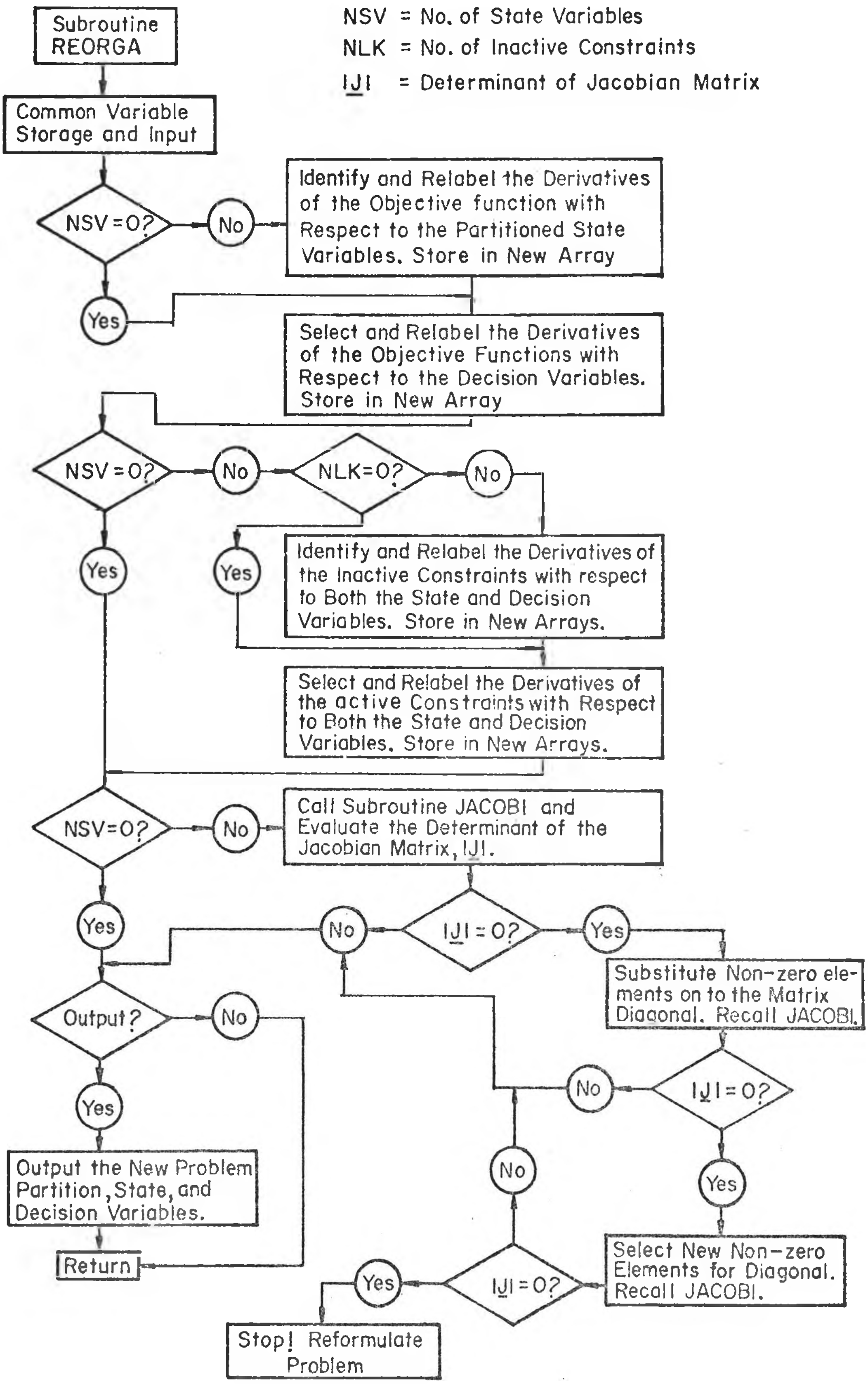


Figure 6. Illustrative flow chart of the subroutine REORGA.

determinant is evaluated. If this value is not zero, then REORGA concludes its function and control is returned. However, if for some reason the current partition between states and decisions yields a singular value for the Jacobian matrix, REORGA attempts to restructure the partition into a non-singular condition. In problems with many state variables, this may be an almost impossible requirement because of the enormous number of variable combinations possible. In REORGA, the best plan that could be thought of was one of trying to make all diagonal values in the matrix non-zero. Unfortunately, cases have been found where this is insufficient in which the problem definition needs to be re-evaluated. Generally, the diagonalization will provide a non-singular Jacobian matrix.

Subroutine NEWTSIM

Throughout this differential algorithm, systems of non-linear equations must be solved in order to determine the real values of the state variables. The procedure for accomplishing this is the so-called Newton-Raphson method, which is incorporated in subroutine NEWTSIM.

This procedure is derived by expanding Equation 2 in a Taylor Series and by ignoring the higher order terms:

$$\underline{f}(\underline{x})^{\nu} = \underline{f}(\underline{x})^{o} + (\nabla_x \underline{f})\,\partial \underline{x} \quad . \quad . \quad . \quad . \quad . \qquad (32)$$

Then, noting that $\partial \underline{x}$ can be approximated by $x^{\nu} - x^{o}$, and rearranging terms:

$$\underline{x}^{\nu} = \underline{x}^{o} - (\nabla_{\underline{x}}\underline{f})^{-1}\ \underline{f}(\underline{x})^{o} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (33)$$

This recursive equation can then be used to solve the nonlinear equations.

There are several problems with the Newton-Raphson method which demand attention in NEWTSIM. Occasionally, the system of equations being solved represent functions with several inflection points or nodules. In these situations, if the step size in a decision variable is too large, the procedure may converge on meaningless points. To combat this occurrence (which is often), the NEWTSIM subroutine is able to back up until a proper solution is obtained. An illustrative flow chart of this subroutine is shown in Figure 7. In some cases, the procedure will not converge on a solution. Generally, this means a poor problem formulation, but if it occurs, the output subroutines are called and the program will stop.

Subroutine DECDJ

DECDJ is the subroutine in which the particular decision variable, $d_p$, is decreased. It, along with INCDJ and INCFT, is the basic component in this optimizing method, and it is by far the most complex. This subroutine has been flowcharted in Figure 8.

The first operation of DECDJ is to store the entering values of the decision variable, $d_p$, and the constrained derivative of the objective function, $\delta y/\delta d_p$. Then, subroutines calculating the constrained derivatives, $\delta s_i/\delta d_p$

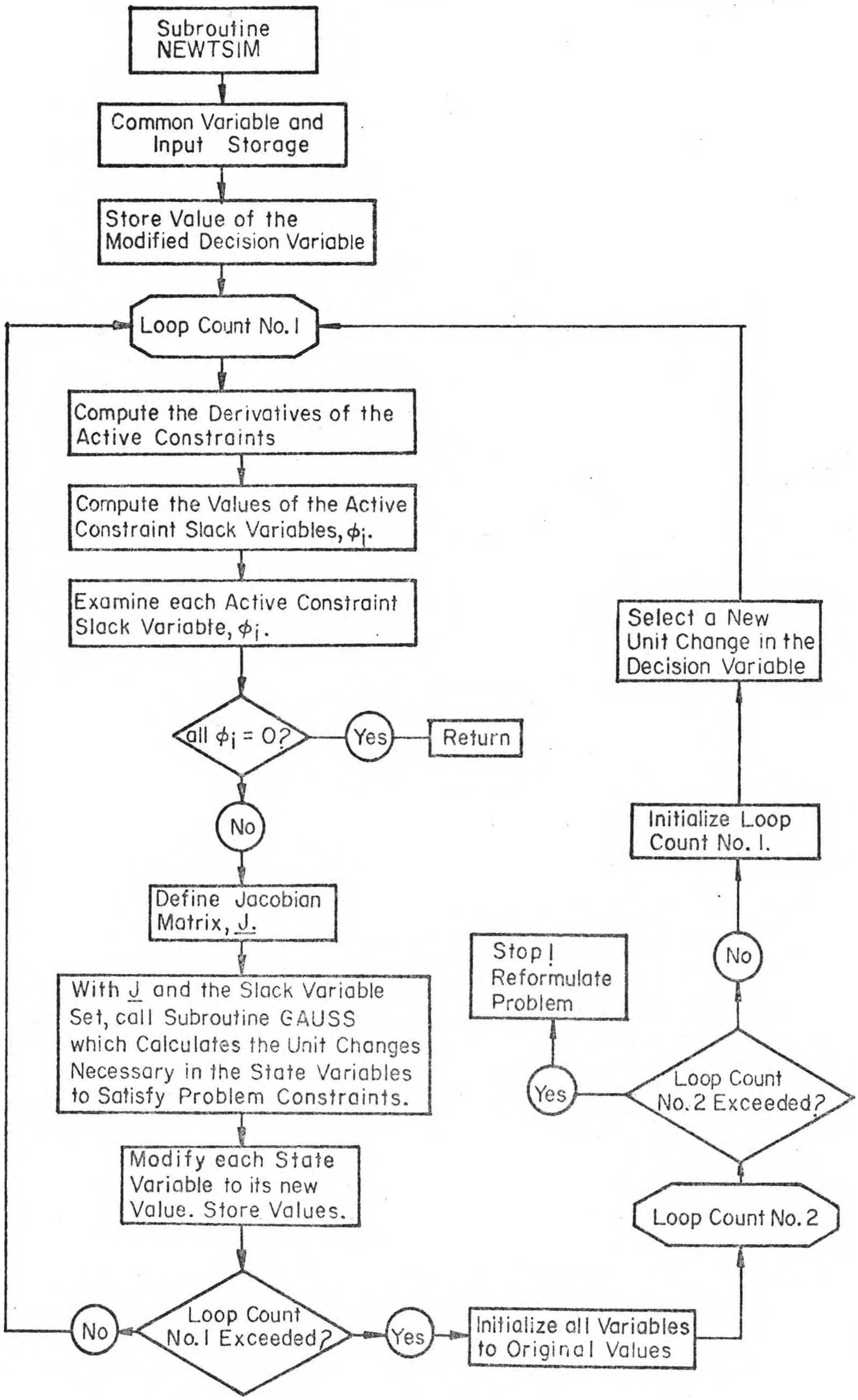


Figure 7. Flow chart of the subroutine NEWTSIM used to solve systems of non-linear equations.

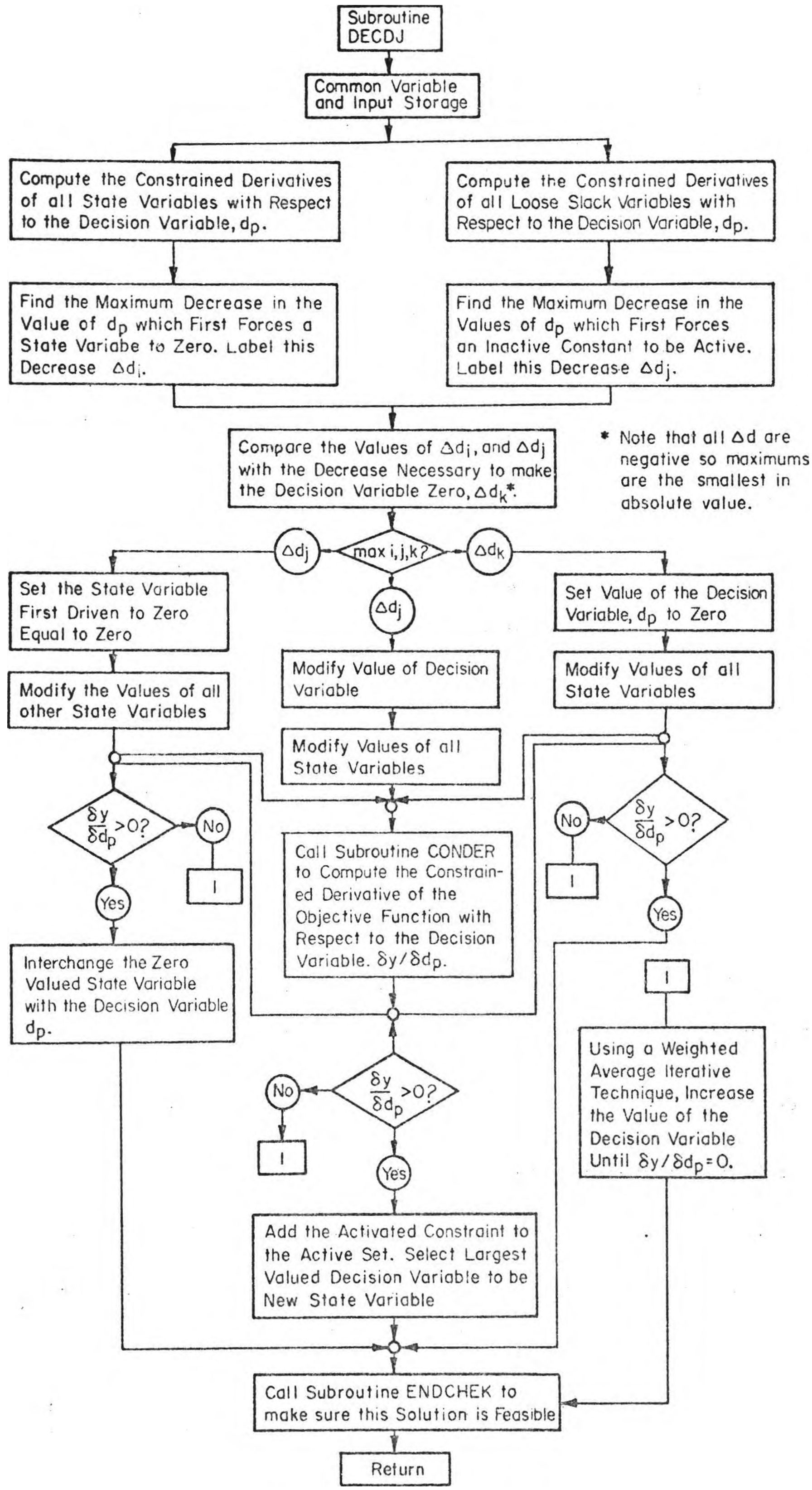


Figure 8. Illustrative flow chart of the subroutine DECDJ.

and $\delta f_{\ell}^{+}/\delta d_p$ are called. Next, the most limiting condition affecting the magnitude of the decrease in $d_p$ is evaluated. Based upon this determination, the appropriate change in the decision variables is made and a new value for $\delta y/\delta d_p$ is calculated. If this value has become negative, the decrease has been too large and the procedure progressed past the minimum. When this occurs, the initial and new values of $d_p$ and $\delta y/\delta d_p$ are used for a weighted average iterative procedure to adjust $d_p$ to a value that results in $\delta y/\delta d_p$ being equal to zero. On the other hand, if these constrained derivatives of the objective function remain positive, the program continues. Three conditions occur:

1. $d_p$ can be decreased to zero and no changes in the problem structure are necessary.
2. The decrease in the decision variable can force a state variable to zero. The particular state going to zero is already known, so the largest valued decision variable is interchanged with the state. Then, with the old state variable equal to zero, the new set of equations can be solved.
3. The decrease in $d_p$ may result in a previously inactive constraint being tightened. In this situation, the number of state variables must be increased by one and a new variable and constraint partition determined.

At the conclusion of these adjustments, the subroutine ENDCHEK is called to make sure the new problem structure is a realistic one. If so, the control is passed first back to DECDJ and then to DIFALGO for a new iteration. If not, ENDCHEK redefines the structure and partition until they are satisfactory.

# Chapter 4

# URBAN WASTEWATER AND RECLAMATION MODEL

## Introduction

The urban wastewater treatment and reclamation system is a complex network of unit operations, flow control points, and water quality objectives. Associated with each unit of treatment are the capital costs of construction and the costs of operating and maintaining these facilities. An analysis of these costs by Dérédec (1972) indicates that these facilities exhibit significant economies of scale, i.e., the marginal costs decrease with capacity. In a review of several sources of information, Dérédec (1972) summarized the costs of these facilities into useable cost functions and then compares the predicted values using these relationships to actual installations. These results indicated an accuracy of within about 10-20%. This accuracy is also sufficient for the purposes of this investigation.

In the model of the wastewater treatment system developed in this chapter, these relationships are used to reflect the costs of treating and reclaiming wastewater for recycling and achieving the standards set for urban effluents.

## Formulation of Wastewater Treatment Model

The intent of the wastewater treatment model, illustrated in Figure 9, is to minimize the costs of the facilities subject to the water quality standards placed on the urban effluent and the water being recycled. The costs of recycled water are determined as the unit difference between the total system costs with and without recycling. Thus, by dividing the difference in these costs by the quantity of water to be reused, an average cost, or unit cost, for this water can be determined. The optimization of the wastewater treatment system minimizes the unit costs of recycled water, as well as the costs of achieving certain levels of pollutants in the released effluent.

For the purposes of this study, the water quality vector will be limited to two parameters: (1) the inorganic concentration of total dissolved solids, TDS; and (2) the commonly cited 5-day Biochemical Oxygen Demand, BOD. However, the cost functions represent treatment facilities which remove suspended solids, nitrates, phosphates, and other pollutants restricted by water pollution guidelines, as set by the regulatory agencies. The consideration of only two of these parameters by no means assumes that other quality criteria are unimportant. Instead, the intent of this limitation is to select two parameters that best characterize the overall quality of water. The evaluation of water management policies in the

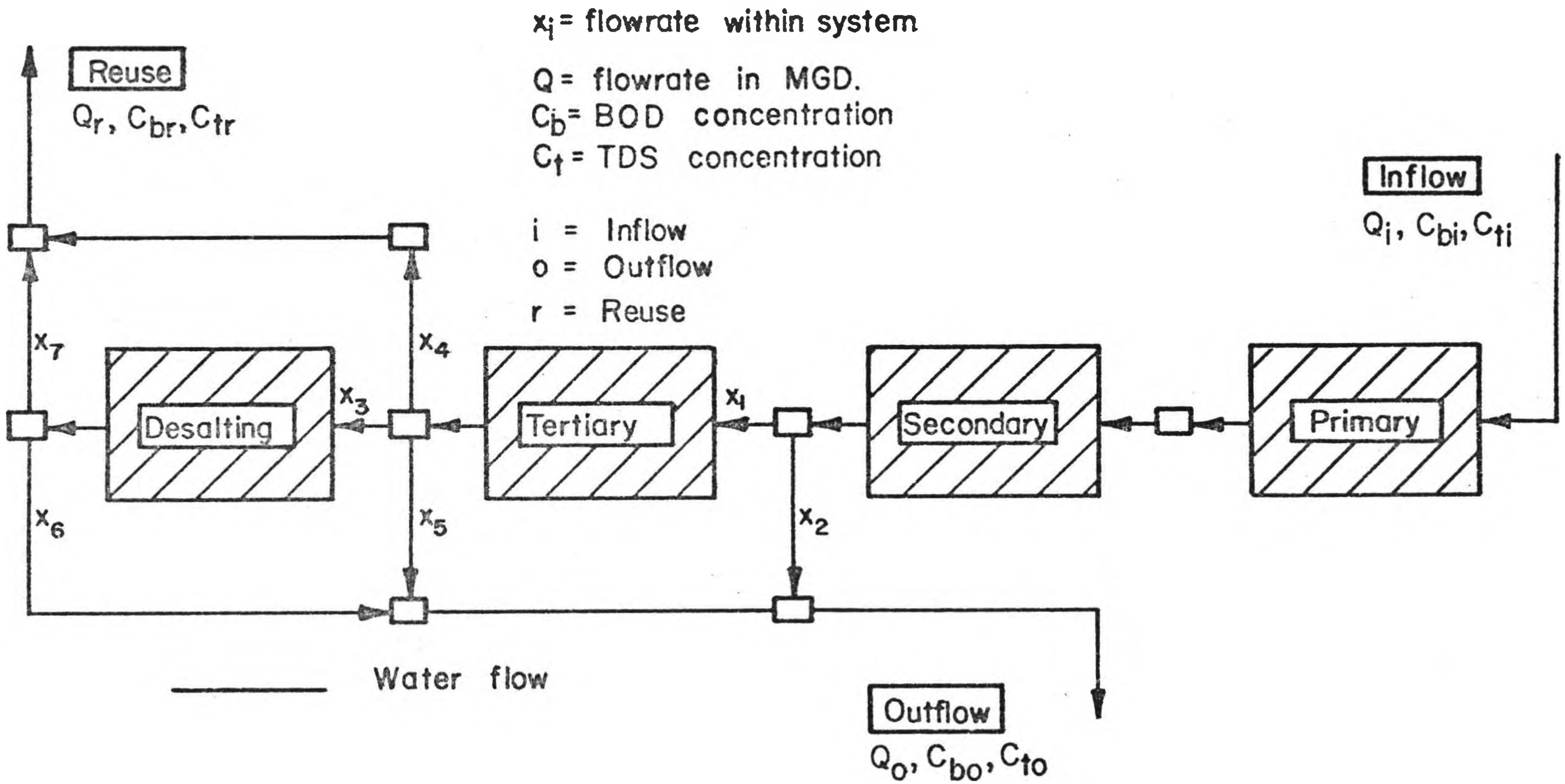


Figure 9. Schematic flow network of an urban wastewater treatment system.

urban environment requires that the interdependence between the sectors of the model be properly defined. As a result, TDS and BOD were selected as "indicators" of the effects that water quality in one part of the model have on the others. These variables are also widely used in design and monitoring and therefore are commonly measured.

Wastewater from the urban area is collected and sent first to the primary treatment. The quantity of these flows is defined as $Q_i$ while their associated TDS and BOD concentrations are $C_{ti}$ and $C_{bi}$, respectively. The primary effluent then becomes the influent to the secondary treatment phase. Upon concluding secondary treatment, the BOD levels are usually low enough to satisfy the 80% removal specified by present water quality standards (Nichols, Skogerboe, and Ward, 1972). However, as the water quality standards become more rigid, further treatment is necessary. Consequently, a decision must be made at this point as to how much water should be spilled into the effluent channels, $X_2$, and how much should be sent through tertiary treatment, $X_1$, in order to achieve a mix with a given level of BOD in the final urban effluent, $C_{bo}$. After tertiary treatment, three additional flow parameters must be decided upon: (1) the quantity of water released to the outflow, $X_5$, (2) the quantities released to the reuse system, $X_4$, and (3) the flows needing desalinization, $X_3$, to satisfy specified levels of TDS in both the outflow and the recycled water. The TDS constraints on the reuse system and outflows are defined as $C_{tr}$ and $C_{to}$, respectively, which are met by mixing flows passing through the desalting process ($X_6$ and $X_7$), with the other flows. The flows in the wastewater model are regulated according to the water quality standards, physical system at each junction, and the quantities of outflow, $Q_o$, and reuse, $Q_r$.

The water quality objectives in this model function as constraints on the optimization procedure. Two constraints on the effluent water quality thus describe the restrictions on the two quality parameters, TDS and BOD. The functions can be written as,

$$X_2T_1 + (X_5 + X_6)C_{br} \le Q_oC_{bo} \quad . \; . \; . \; . \; . \; . \; . \quad (34)$$

and,

$$(X_2 + X_5)C_{ti} + X_6T_2C_{ti} \le Q_oC_{to} \quad . \; . \; . \; . \; . \; . \quad (35)$$

in which $T$ is the BOD concentration after secondary treatment in mg/l, $T$ is the removal efficiency for the desalting process, in mg/l and $C_{br}$ is the BOD concentration from tertiary treatment. The water quality constraints for the reuse segment can be written as,

$$X_4C_{ti} + X_7T_2C_{ti} \le Q_rC_{tr} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (36)$$

representing only the concentrations of TDS since it is practical to assume that BOD levels after tertiary treatment would generally satisfy criteria for raw water supplies.

The interaction of flow rates and water quality, extends the mathematical non-linearity to the constraints of the preceding paragraph. Therefore, it is necessary to add physical flow constraints to the model to avoid unusual flows in the network. To begin, consider the outflow,

$$X_2 + X_5 + X_6 = Q_o \quad . . . . . . . . . . . . . \quad (37)$$

and also the reuse phase:

$$X_4 + X_7 = Q_r \quad . . . . . . . . . . . . \quad (38)$$

In addition, the flow system must also be feasible at each decision junction in the model:

$$X_1 + X_2 = Q_i \quad . . . . . . . . . . . . \quad (39)$$

$$X_1 - X_3 - X_4 - X_5 = 0 \quad . . . . . . . . . . \quad (40)$$

$$X_3 - X_6 - X_7 = 0 \quad . . . . . . . . . . . . \quad (41)$$

The cost functions for each treatment process, along with these constraints, form the optimizing model of the wastewater treatment system.

## Model Components

### Primary Treatment

The first component of urban wastewater renovation, primary treatment, consists primarily of screening, grit removal, and primary clarification. Although these processes are quite often incorporated with secondary treatment, sufficient cost information exists in the literature to make the distinction.

Capital construction cost estimates for primary treatment facilities have been reported by several researchers. These estimating functions are helpful not only in establishing the costs of water quality control, but also in the planning of treatment plants themselves. Typically, these relationships have the form,

$$Y = aZ^{m} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

in which Y is the capital cost in millions of dollars and Z is the plant capacity in million gallons per day (mgd). Primary treatment as a whole is relatively subject to economics of scale as the exponential coefficient, m, usually ranges between 0.7 to 0.6. Smith (1968) states that in terms of 1967 dollars,

$$Y = 0.316Z^{0.71} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

while Shah and Reid (1970) propose a 1959 dollar value expression of:

$$Y = 0.331Z^{0.61} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

The operation and maintenance costs have also been of interest to managers, builders, and planners of wastewater treatment systems. The formulas which have been proposed by several investigators have the same general format as expressed in Equation 42. Michel (1970), for example, indicates that in 1967 dollars, the operation and maintenance costs are,

$$Y_o = 21{,}880Q^{0.59} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

where $Y_o$ is the total annual operation and maintenance costs and Q is the average daily flow in mgd. However, these costs are more commonly expressed as costs per 1000 gallons treated, such as the work by Smith (1968) which uses 1967 dollars,

$$y = 4.47Z^{-0.17} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (46)$$

in which y is the operation and maintenance costs in cents per 1000 gallons.

Secondary Treatment

The principal components of the secondary treatment are most commonly either activiated sludge, rapid rate trickling filters, or slow rate trickling filters. For the purpose of this writing, the activated sludge process was selected primarily for its flexibility with respect to varying removal efficiencies. Although activated sludge appears to achieve greater removal efficiencies and more design flexibility than the other two, the costs are also somewhat higher. In a design context, the respective choice would be based on a more comprehensive analysis than is appropriate here.

The capital construction costs of building secondary treatment plants, while not indicating as large an economy with scale as encountered in the primary treatment plants, do nevertheless exhibit costs relationships with declining marginal costs with increased capacity. Shah and Reid (1970) state that in equivalents of 1959 dollars, the capital construction costs for these plants can be estimated from the following relationship,

$$Y = 2.48 \times 10^{-4} \, (PE)^{0.47} \, Z^{0.22} \quad . \; . \; . \; . \; . \; . \; . \quad (47)$$

where PE is the Population Equivalent of the organic loading expressed as,

$$PE = \frac{8.34\ QC_{bi}}{b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (48)$$

in which Q is the average daily flow in mgd, $C_{bi}$ is the Biochemical Oxygen Demand (BOD) concentration of the flows in mg/l, and b is a constant, usually 0.17 lb of BOD per capita per day. Smith (1968) also presents an estimate of activated sludge plant costs:

$$Y = 0.58\ Z^{0.80} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (49)$$

Of some additional interest is Shah and Reids' (1970) estimate of the costs of the activated sludge unit itself. This relationship, having the same format as Equation 47, is given in 1959 dollars as:

$$Y = 5.1 \times 10^{-3}\ PE^{0.46}\ Z^{0.36} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (50)$$

The costs associated with operating and maintaining secondary treatment plants are listed in several sources. For example, Michel (1970) suggests three relationships for these costs:

$$Y_o = 3.16 \times 10^{4}\ Q^{0.73} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (51)$$

$$Y_o = 28.2\ PE^{0.75} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (52)$$

$$y = 9.02\ Z^{-0.107} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (53)$$

Tertiary Treatment

Advances in wastewater treatment have led to several demonstrations of the feasibility of adding tertiary treatment to existing primary, secondary treatment facilities for further removal of waterborne contaminants (Evans and Wilson, 1972). Such advances have been prompted by several mounting crises. First, the need for water by municipalities, industries, and agriculture is outstripping the supplies of natural waters under existing inefficient practices. Secondly, 90% removal of BOD, for example, is not considered satisfactory to continually insure public safety and palatability (Fair, Geyer, and Okun, 1968). And finally, the advent of new pollutants in conjunction with mans' increasing life span subjects him to extended exposures to chemicals with yet uncertain results (Civil Engineering, 1972). Since very few if any existing primary-secondary wastewater treatment facilities can meet reduced pollution levels, or support a "zero level pollution" philosophy for urban effluents, tertiary treatment will become as necessary in the near future as secondary treatment is now.

In this writing, tertiary treatment will consist of flocculation, lime treatment, and sedimentation; granular carbon adsorption; and ammonia stripping. Although tertiary treatment has found only limited application to date, sufficient testing has been completed to generate general cost functions.

The capital costs of tertiary treatment can be found in several sources. Smith (1968) suggests that in terms of 1967 dollars, the following relationships can be employed:

$$Y = 0.05\ Z^{0.89} \quad \ldots\ldots\ldots\ldots \quad (54)$$

for flocculation, lime treatment, and sedimentation,

$$Y = .398\ Z^{0.65} \quad \ldots\ldots\ldots\ldots \quad (55)$$

for grannular carbon adsorption and,

$$Y = .0398\ Z^{0.90} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (56)$$

for ammonia stripping. Barnard and Eckenfelder (1970) also give an estimating formula for granular carbon adsorption in 1959 dollars which will be included here for comparison:

$$Y = 0.20\ Z^{0.86} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (57)$$

The costs of operating and maintaining tertiary plants are also listed by Smith (1968) in terms of 1967 dollars:

$$y = 2.99\ Z^{-0.038} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (58)$$

for flocculation, lime treatment and sedimentation,

$$y = 10\ Z^{-0.28} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (59)$$

for granular carbon adsorption and,

$$\begin{aligned} y &= 11.58\ Z^{-0.3} \qquad Z \leq 3 \text{ mgd} \\ y &= 1.2\ Z^{-0.04} \qquad Z > 3 \text{ mgd} \end{aligned} \quad . \ . \ . \ . \ . \ . \ . \quad (60)$$

for ammonia stripping.

## Desalting

The removal of salts from seawater and brackish waters has been under close examination for some time as a source for supplemental water supplies (White, 1971). The limiting factor to date has been the high costs as compared to other water sources. Among the promising techniques that have been developed, either electrodialysis, reverse osmosis, or a combination of these two methods seems to be the best suited for reclamation of urban wastewater (Dykstra, 1968). Again the flexibility with regards to removal efficiencies prompted the selection of electrodialysis for this model.

The capital construction costs for desalting plants has been suggested by Smith (1968) to be,

$$Y = 0.51\ Z^{0.67} \qquad (61)$$

and by Rambow (Cited by Dérédec, 1972),

$$Y = 0.219\ Z^{0.66} \qquad (62)$$

which are also in terms of 1967 dollars. The first equation is for a 90% TDS removal and the second is for a removal of 500 mg/l.

The same two sources supplying capital cost information also suggest the following operation and maintenance costs:

Smith (1968) $\quad y = 47.94\ Z^{-.21} \qquad (63)$

Rambow $\quad y = 10.2\ Z^{-0.12} \qquad (64)$

indicating significant variation.

A single-stage electrodialysis process applied in this model is assumed to have a removal efficiency of about 40%. Therefore, the costs suggested by Smith (1968) would be for a four-stage demineralization system.

## Operation of Wastewater Treatment Model

In order to provide the reader with a clearer understanding of the urban wastewater treatment model and illustrate its use in evaluating optimal policies in the overall urban water system, it is useful to examine some of the types of results generated by the wastewater treatment model.

The cost functions presented in Equations 34 to 64 are in the present-worth dollar value set by the original authors. These relationships were multiplied by an adjustment factor to convert all of them to 1970 dollar values and then a set of results were generated to delineate the basic characteristics of the system.

The first characteristic of interest is the effects varying effluent quality standards have on the unit costs of recycled water. An illustration of this influence, shown in Figure 10, represents a system with a reuse capacity ($Q_r$) of 30 mgd, and a fixed effluent TDS standard ($C_{to}$) of 600 mg/l. The unit costs (present-worth) are shown as a function of BOD standards on the outflows ($C_{bo}$) with four levels of TDS concentrations in the recycled flows. It is interesting to observe the curves at an abcissa value of about 10 mg/l. At this point, the unit costs for the higher limits of TDS in the reuse become negative. This characteristic illustrates that when water quality standards on the outflow are sufficiently restrictive, the capacity of the desalting plant (and therefore the costs) is larger than if the system would permit some flows to be diverted to reuse at a poorer quality. The effect therefore of increasingly stringent standards on urban effluents is to substantially enhance the feasibility of reclaiming and reusing wastewaters.

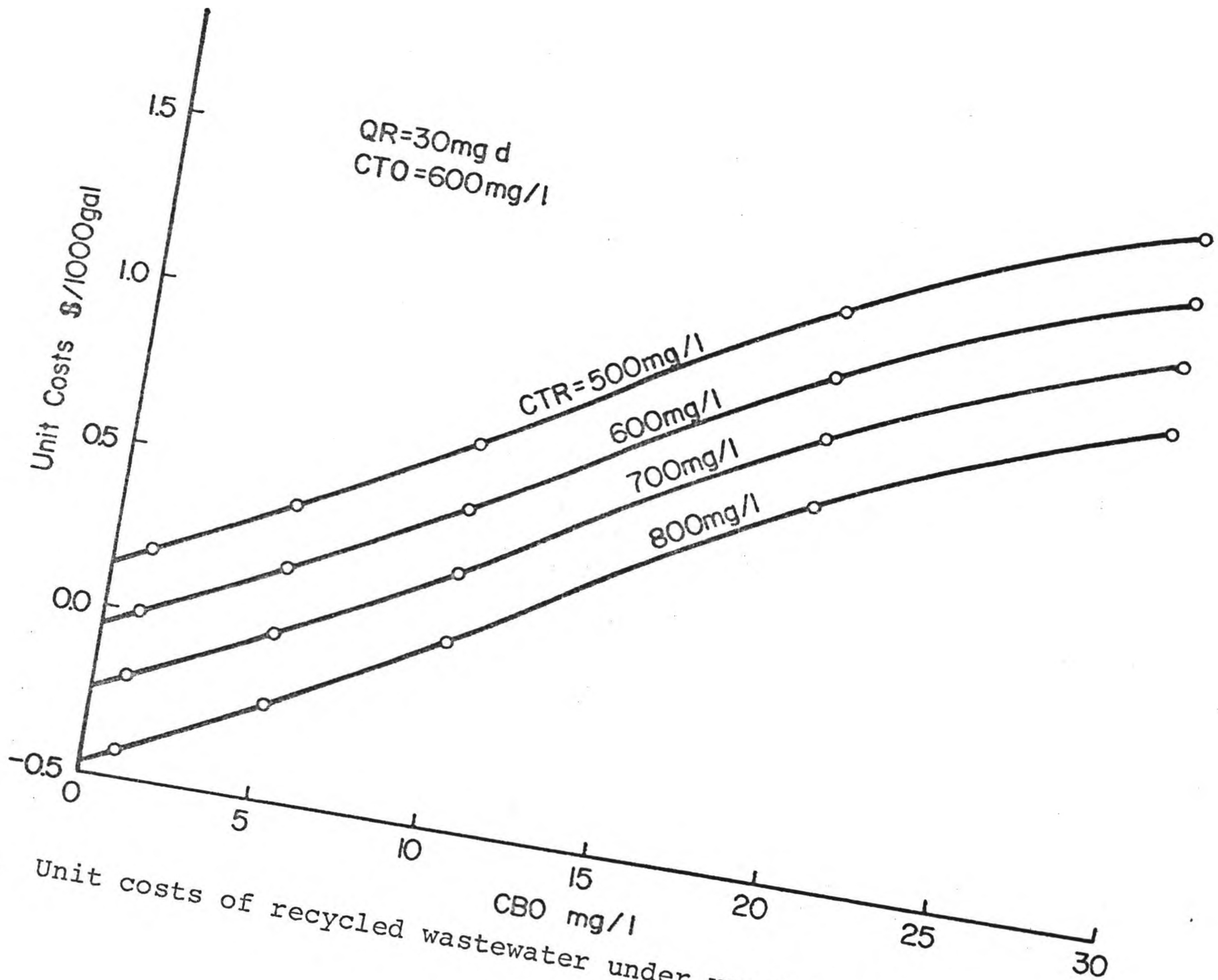


Figure 10. Unit costs of recycled wastewater under varying water quality standards.

Another interesting attribute of the wastewater treatment model is shown in the plots of Figure 11. The economics of scale in the reuse system are shown to be affected by the water quality constraints on the outflow. Results from two analyses in which the outflow BOD concentration is restricted to a value of 10 mg/l are plotted. In the upper segment, the TDS standard is fixed at a value of 800 mg/l and the unit costs for a series of reuse capacities are computed, as functions of the concentrations of TDS in the recycled water. It is observable that larger capacities are much less affected by the level of TDS in the reuse than are the smaller values. Furthermore, the economy of scale is clearly evident with the larger systems having unit costs that are substantially less at the lower concentrations of TDS in the recycled water. The curves in the lower segment have the same basic characteristics as the upper curves, except that the outflow TDS standard is set at 500 mg/l. It is interesting to note that the scale effects are almost eleminated.

Each of the Figures 10 and 11 demonstrate the impact that increasingly rigid water quality standards have on the economic feasibility of reusing some urban effluents as supplemental water supplies. The wastewater treatment model discussed thus far in this chapter is employed in the overall model to optimize the water supply policies in the urban water supply and distribution segments. However, aside from the water quality constraints imposed on the

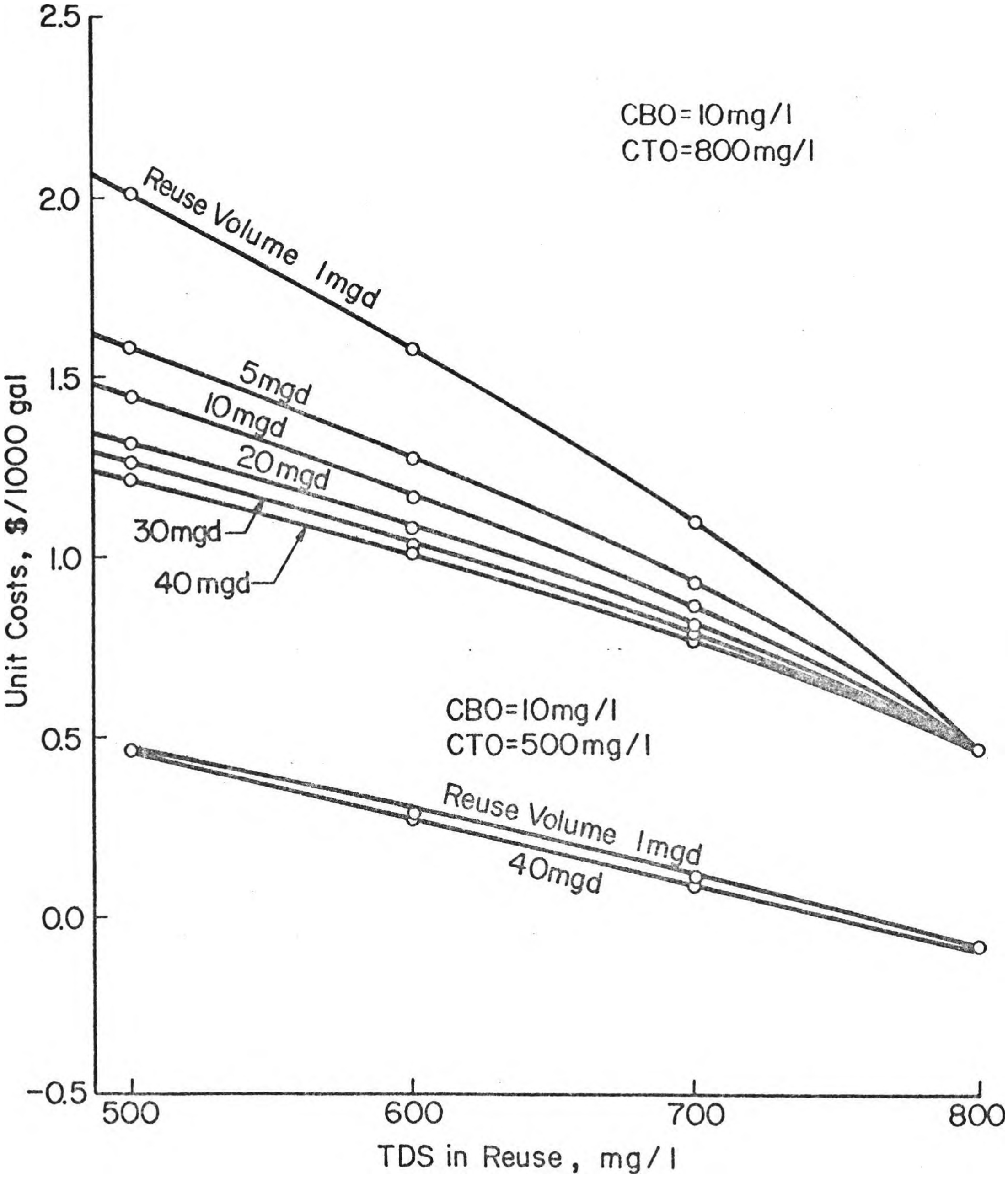


Figure 11. Effects of varying water quality standards on the unit costs and economies of scale of recycled wastewater.

urban effluent, the optimal values of reuse, its TDS concentrations, and several other important variables are not known during the initial stages of the problem solution. Consequently, the context of decomposition is used to iteratively improve the solution until the optimal policy is formulated. To do this efficiently, only one curve in Figure 11 is generated and a polynomial regression of the function is calculated. This is done initially for assumed variable values in the wastewater treatment system. Then the water supply and distribution model is optimized. New values of input to the wastewater treatment model are now known and another iteration is made. This process is repeated until no further refinement is possible.

Until this point, all the cost functions have been in terms of total present-worth. However, in the examination of the optimal water management policies, annual costs are more commonly employed. To facilitate these requirements, the present-worth calcualtions in the preceding paragraphs have been transformed into a uniform series of annual costs. To do this, it has also been necessary to add the interest costs to the function. This procedure has been accomplished in Figures 12 and 13 in order to demonstrate the final results gained from this submodel.

Before proceeding with a description of the water supply and distribution model, some comment regarding the assumptions made to formulate the wastewater treatment model should be made. First, no attempt has been made to model

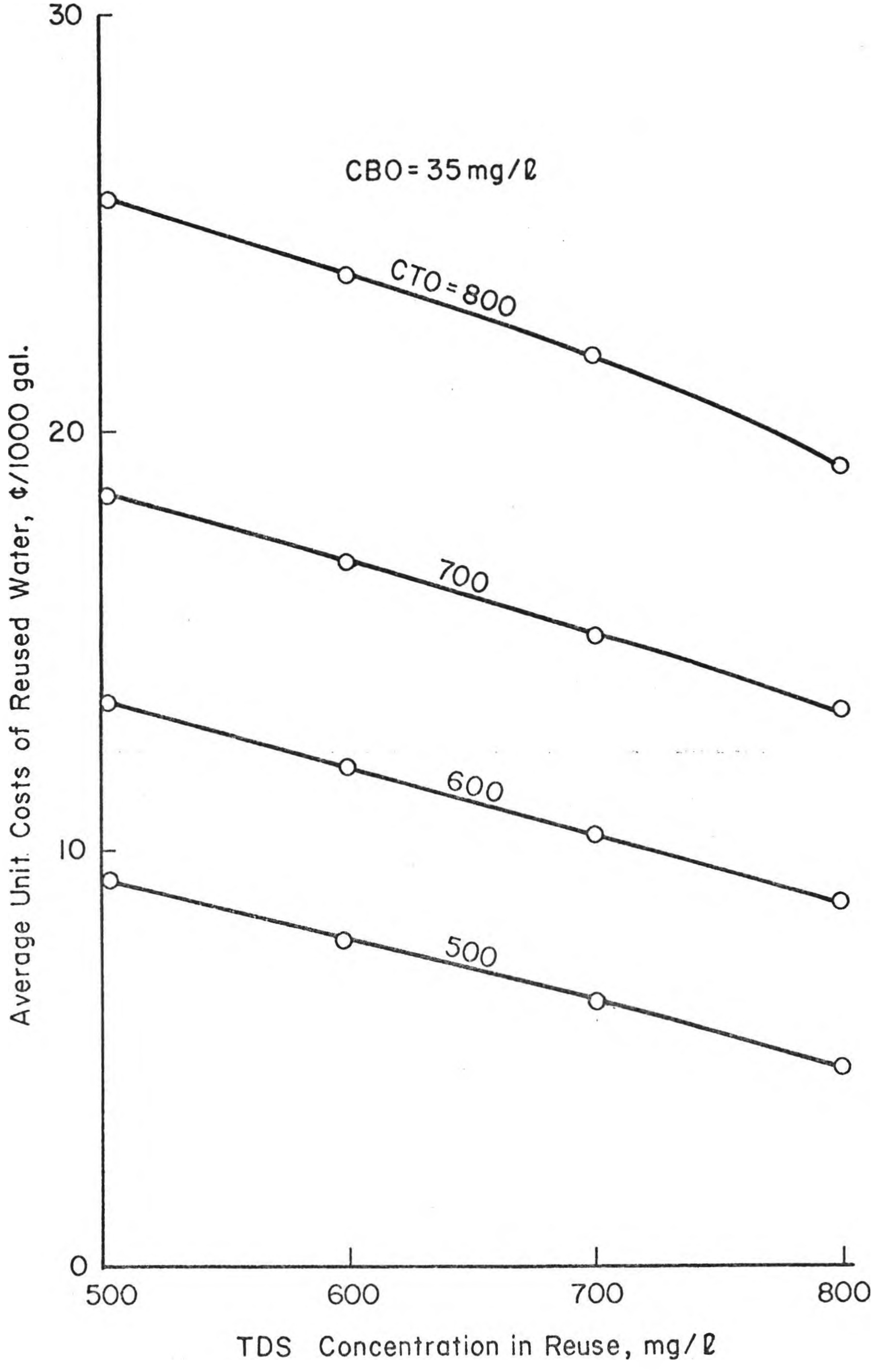


Figure 12. Average unit costs for reused wastewater for a BOD limit of the urban effluent, CBO, of 35 mg/l and various levels of effluent TDS, CTO.

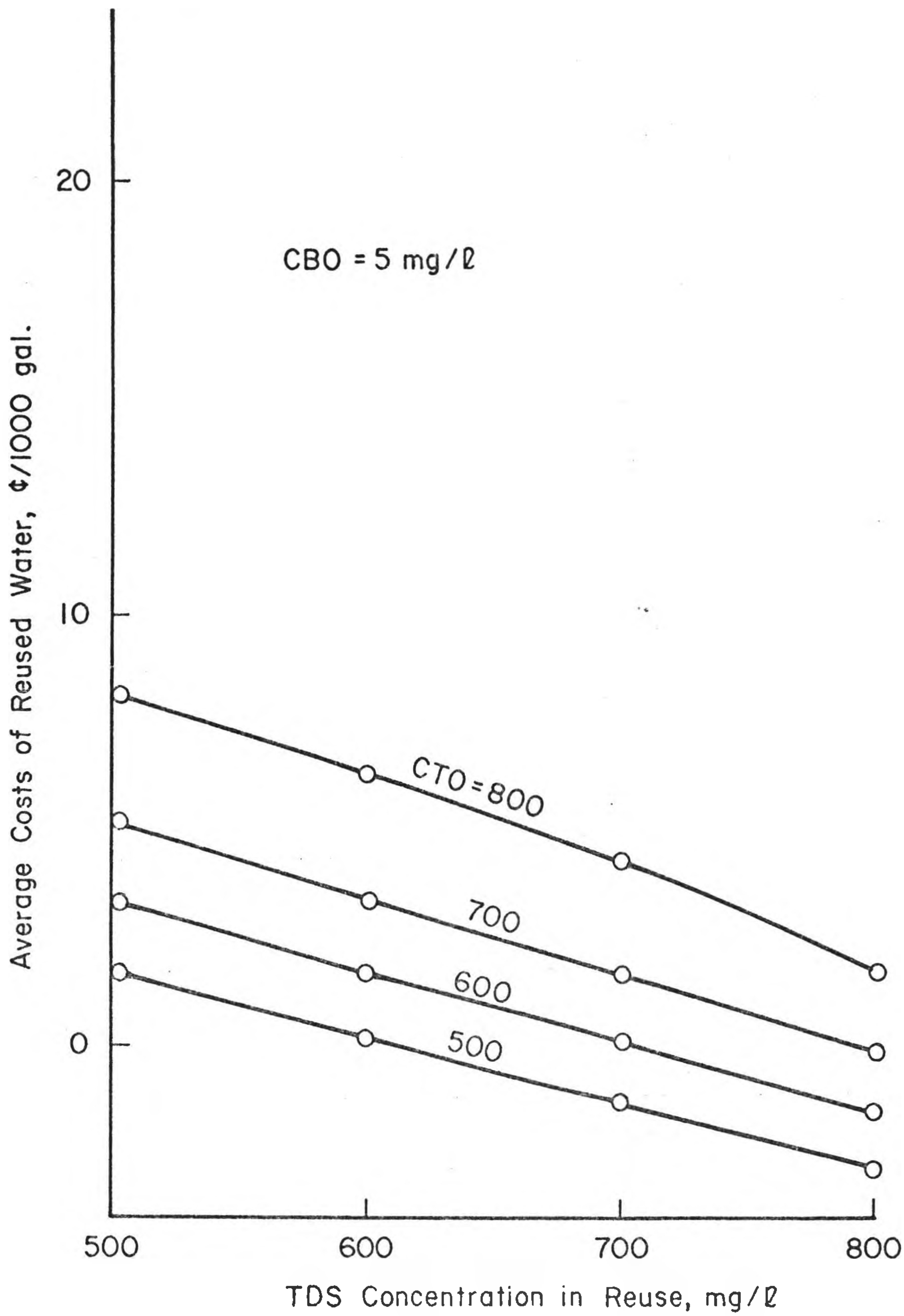


Figure 13. Average unit costs for reused wastewater for a BOD limit on the urban effluent, CBO, of 5 mg/l and various levels of effluent TDS, CTO.

the numerous alternative treatment schemes which may improve the cost-effectiveness of this system. For example, the use of polymers in primary treatment have been shown to significantly improve the primary removal efficiency in certain circumstances (Henningson, et. al, 1970). Justification for this assumption is because cost information is not accessible at this time in the general literature from which cost functions could be formulated for inclusion in this section.

# Chapter 5

# THE URBAN WATER SYSTEM MODEL

## Introduction

In the preceding chapter, the urban wastewater treatment and reclamation system model was developed prepatory to its use here. The water supply and distribution system model, defined in this section, focuses on the analysis of water supply alternatives. However, because recycling is an integral part of urban water supply, the linkage between the two systems is brought to light.

The scope and format of the urban water system model derived and explained in this chapter follows the "limited purpose model" concept. The intent of this development is to provide the mathematical description of the broad and macroscopic characteristics of urban water systems and evaluate the effects of changing institutional constraints, such as water quality goals, on the optimal water management policies. Consequently, the model is less useful as a design or capacity determining tool as it is for delineating and comparing various planning and management alternatives. By limiting the scope of the model in this manner, and avoiding the entangling detail of the exact nature of the flow networks, the model can be general in nature and adopted to other areas with a minimum of modification.

The basic nature of the model as it is operated begins by combining the water supply alternatives with the distribution system, leading to the individual urban demands, as illustrated in Figure 14. This leaves the urban water system model in two distinct components which are then conjunctively solved to optimize the complete network. The procedure involves the following three steps:

(1) A quantity and quality of water needed for reuse from the reclaimed wastewater treatment system is assumed and unit costs for this water are computed for a range of TDS concentrations. Then a polynomial regression of these data points is computed giving the unit costs of recycled water as a function of its TDS concentrations.

(2) Using the unit costs previously determined for recycled water, the model optimizes the water supply and distribution subsystem to evaluate the optimal water management policy.

(3) The assumed values of reuse are contrasted with the quantities actually employed in the optimal plan. If these values differ markedly, new values of reuse parameters are assumed and the process repeated.

In addition to recycling wastewater, alternative water supplies such as interbasin water transfers,

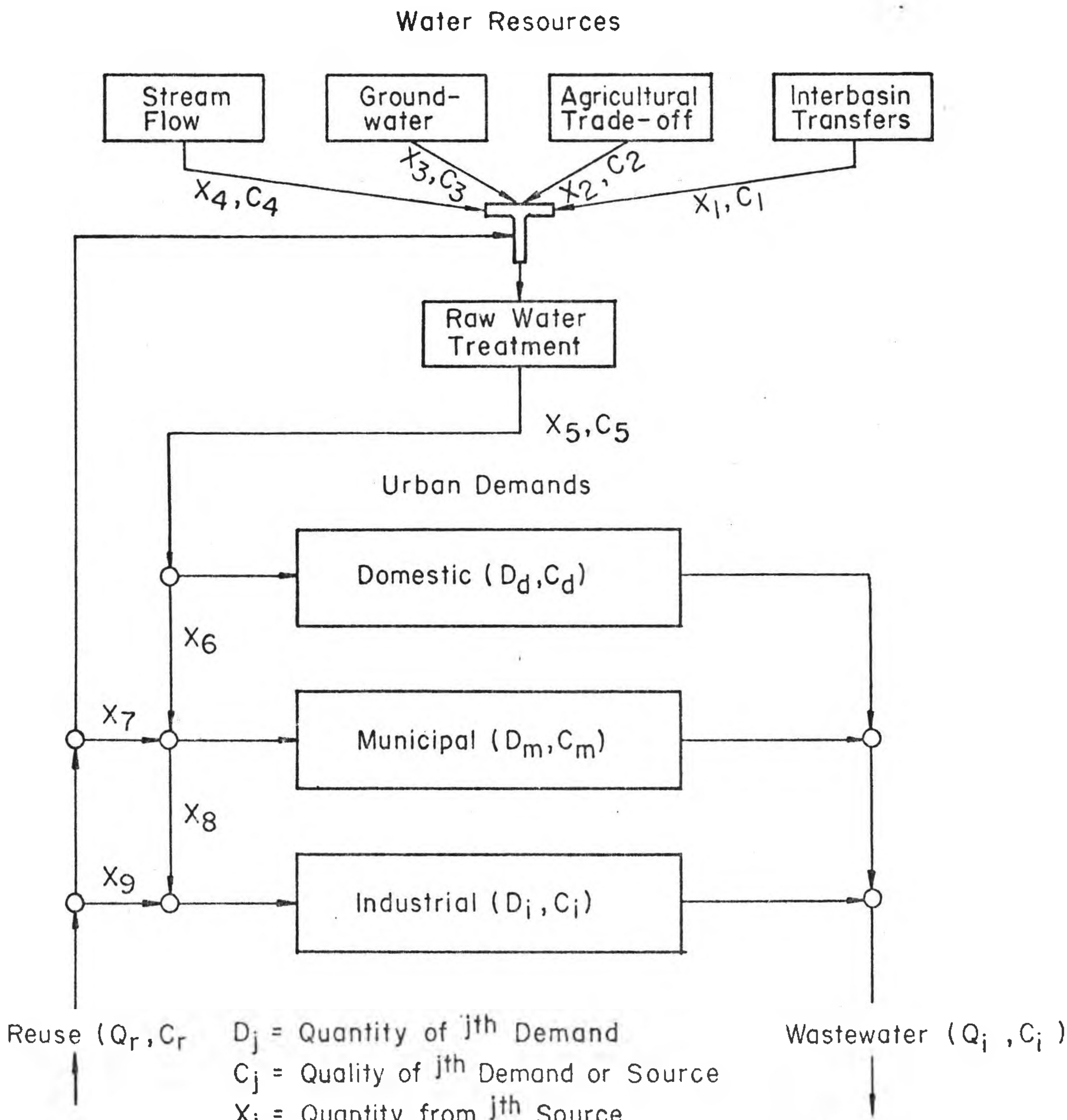


Figure 14. Shcematic diagram for the urban water supply and distribution system model.

groundwater, agricultural right aquisition and transfer, and, of course, in-basin stream flows are available. The characteristics of these resources are complex, but have been extensively investigated. Although a thorough review of such characteristics is infeasible, some general comments are helpful in realizing the potential uses and extensions of the urban water system model.

## Water Sources

### Stream Flows

Water resources generated within the hydrologic unit encompassing the urban demand consist primarily of stream flows. Reservoirs, diversion works, and conveyance systems transform the stochastic variations in these flows into demand frequency water supplies. Such supplies were developed in competition with other interests, such as the agricultural and mining enterprises. Because urban areas have generally been junior appropriators, they have been forced to develop additional alternative water sources to meet growing needs.

The competitive characteristic of over-appropriated water supplies has prompted numerous attempts to impart an economic value or price to the flows. Such studies seem to indicate that water values are higher than existing prices because of the protective influence of the water right system. In order to avoid the surrounding controversy regarding the planning values these resources should assume, existing rights are valued at estimated current costs. Then, in the model operation to be described later, an attempt is made to indirectly value additional stream flows according to actual worth. A more specific description of this analysis will be deferred until later in this writing.

## Interbasin Transfers

Transferring water resources from one river basin to another is among the most feasible alternatives for supplying the needs of water-short areas. However, most proposals for these transfers in the West have resulted in political conflict because of the high value of the scarce resource. Such conflicts have been observed in both the Congress and State legislatures, in the Federal agencies, in the association of Federal and State governments, in the courts, and even on the canal or reservoir banks (National Water Commission, 1972). The conflicts have also extended to various interest groups, such as conservationists and the large urban centers.

The costs of interbasin transfers are three dimensional in nature. First, the capital outlays for construction of the necessary facilities to import the water are very large. However, unlike the water treatment plants, these facilities are relatively permanent in nature. For example, the structures involved may include reservoirs, tunnels, canals, and diversion and control structures. Since the expected life of these facilities extend beyond the planning horizon, they may be easily financed for as much as 50 years or more, which substantially reduces the annual costs. The second economic consideration regarding interbasin transfers is the operation and maintenance costs. Again, the permanent nature of the system is characterized by a relative freedom from maintenance and thus have low operation and maintenance costs. And finally, the third dimension of costs associated with water importation is the externalities, or costs to downstream water users as a result of the water transfer. Externalities are difficult to quantify and require a more specific analysis than is permissible here.

External costs cannot be ignored in regional water resource management, but are difficult to incorporate into local planning efforts. It would appear plausible therefore, to suggest further investigations regarding the external effects of the importations and coordinate water developments on a state level.

Because interbasin transfers are being considered on such a large scale in the western states, it would be helpful for the cost versus capacity relationship to be incorporated in a model such as this. However, such an analysis has not heretofore been completed in sufficient detail to include in this description. Thus, the model as presently formulated uses only unit costs based on data available in the various project reports.

## Agricultural Water Transfers

Once a pattern of use has been initiated in a river basin, changes in water usage at one point in the system are reflected elsewhere in the system. Such changes may diminish both the quantity and quality of the water resource available to other users. In many cases, these factors may not be critical during normal years because even though the annual flow is completely appropriated, it may not be fully utilized. However, during low-flow periods, or as the demands increase, the damages resulting from changes in use practices may become significant.

As these general trends continue, urban growth may expand the demands for water beyond the safe annual yield of the urban areas' water rights. Historically, two alternatives were immediately obvious: (1) interbasin transfers; and (2) agricultural water right acquisition. Initially, the second alternative was pursued. Cities began buying agricultural water rights and then filing for a change in the points of diversion, thus initiating the transfer of waters within the basin. The attempts to do so along the eastern slope of Colorado were almost futile. In fact, Hartman and Seastone (1970) found in an examination of records in the State Engineer's office, that only 33 cases involving transfer of 22 second-feet from agricultural to municipal use have been successfully completed in the state of Colorado (excluding Denver). Of these, only 9 have been completed since 1930. The poor success in transferring water from agricultural to municipal use according to Hartman and Seastone (1970) is the uncertainty of court interpretations. As a result, the major cities in eastern Colorado have turned primarily to interbasin transfers.

Although a brief discussion of the process of transferring agricultural water to municipal use has been presented, it serves only to justify the approach taken in the urban water system model presented herein. It is concluded that attempting to directly incorporate this alternative in the model would be infeasible because of its largely institutional nature. As a result, water obtained from agricultural transfers will be omitted from the model, but an analysis will be presented which indicates the value of such transfers to the urban users. In this manner, an estimate of the costs of this alternative as a potential water supply can be generated.

## Groundwater

The analysis of groundwater is an important part of total water resource management. Yet, this water resource has undoubtedly one of the weakest institutional structures for optimal management of any phase of the developed hydrologic cycle. Nevertheless, the investigation of water as it exists in subsurface strata has been extensive. In fact, the topic of interest in this study, alternative water supply policies for arid urban areas, is directly related to the exhaustive analysis which has been performed to optimize the conjuctive use of groundwater and surface-water resources.

The use of groundwater for urban water supplies has several distinct advantages over other possibilities. First, the waters located in confined or unconfined aquifers are nearly unaffected by seasonal variations in temperature. Secondly, the relatively slow water movements in these aquifers results in seasonal variations in runoff being almost completely dampened. Thus, the supply remains essentially uniform over a season. Third, groundwater flows are not subject to evaporation generally and outflows are small, so that the groundwater basin acts as a minimal loss reservoir. And finally, water quality characteristics of groundwater tend to be constant with time. Although groundwater pollution is a possibility, usually fresh water supplies when located remain a good source of water supplies.

In this application of the urban water system model, groundwater is not considered alternative because of the unconfined nature of the stream-aquifer system in the South Platte drainage. If application of the model in future investigations needs to include groundwater costs, the necessary changes can be made in the objective function and constraints with little difficulty.

## Water Distribution Network

Within the framework of the urban water use, three broad categories of use exist: (1) domestic uses; (2) municipal uses; and (3) industrial uses (Flack, 1971). Although some areas have incorporated separate distribution systems to serve each of these needs, the general case may only find distinction in rates, seasonal variations, and location. Nevertheless, each of these classifications have individual water quality limits, growth rates affected by independent parameters, and a differing importance relative to the preferences of the local populace.

Albertson, Taylor, and Tucker (1971) also make this separation in the urban water utilization subsystem in order to make their model more amenable to a systems analysis approach in evaluating alternative decisions. However, their primary thrust and that of other writers in the publication, is the need for the systematic approach. This attitude has been adopted in this writing as well. In this section, the purpose is to note a few of the characteristics of each of the three classifications of water use and discuss the assumptions employed in this model as it investigates the various potential decision strategies.

Domestic Water Uses

The first objective of a municipal water supply system is to serve the immediate needs of the population. When water supplies are insufficient to meet all potential demands, one after another of the less important uses may be restricted. In the ultimate limitation, water would only be available for such things as drinking, bathing, and cooking. Thus, the domestic water uses are the first priorities of the urban water system.

It has been traditional to lump outside-the-house uses such as lawn or tree watering with the other domestic uses. However, these uses are also subject to rationing during critical periods. Consequently, lawn and tree watering have been deleted from the domestic category and placed with those considered as municipal uses. Such an assumption may be infeasible due to the physical structure of the distribution system. Nevertheless, it does represent the extreme limit of separation between the two uses, and it is, therefore, of interest in this analysis.

Domestic water uses not only maintain first priority on the water supply, but its water quality as well. Many other uses may be restricted by the concentration of total dissolved solids, suspended solids, phosphates, nitrates, heavey metals, or organisms, to name only a few. Domestic water uses have limits on nearly every water quality parameter currently used. As a result, treatment for such uses must be more extensive. In the model proposed herein, the philosophy has been adopted to maintain water quality limits for the domestic use at their present level, even though these levels may not be as high as the tolerable limits. The basis on which such an assumption is based is the fact that drinking water standards are upper limits and not desirable if reduced water pollutants can be maintained.

Municipal Water Uses

Although the fire protection water needs may be nearly as important in an urban area as the domestic needs from purely a survival viewpoint, other municipal water uses such as park, golf course, lawn, and tree irrigations may be important to the living environment. Such uses are generally regarded as supplemental to the enjoyment of living in the urban area.

A great deal has been said concerning the difficult decisions to be made during periods of water shortage, but the cause of the shortage has not been stated. Certainly a man stranded in a life raft in the middle of the ocean is as water short as another lost in the Sahara Desert. It can therefore be concluded that an important method of extending water supplies by more efficient use would be to rearrange the urban water system in such a manner as to distribute water on the basis of quality needs rather than quantity aspects alone. By dividing the urban water use and extracting municipal uses to be served by a poorer quality water, especially with regards to reuse, more water is available to the needs with more restrictive quality characteristics.

Industrial Water Uses

Industrial water use, or commercial use, is defined somewhat differently in this study than its real meaning. For the purposes of this study, industrial uses are those being filled by the urban water system. Fair, Geyer, and Okun (1968) states that more than 60% of the industrial needs in the United States are met by internal reuse. Thus, only 40% of the industrial requirements are on the average served by the metropolitan water system.

If domestic needs exist because people need water to survive, and municipal needs exist because people demand an enjoyable living environment, then industrial needs exists because people are in the urban area to work in the direct and indirect needs of industry. In other words, people live in cities because of industrial concentration and the needs to support industrial jobs with services. Therefore, the priority on industrial water is second to the domestic uses.

The water quality constraints on industrial water use are as varied as the nature of the industries themselves. In this investigation, industrial quality requirements have been limited to the maximum suggested for public potable supplies.

# Model Formulation

The model proposed in this thesis has been formulated in the context of long range planning, but from a unique viewpoint. By simplifying and generalizing the model beyond the intricate details of either physical or institutional structures of any specific location, two central advantages are gained:

(1) The optimal policies derived and the alternatives evaluated indicate decision making on the scale of planning alternatives of water supplies.

(2) Only those institutional constraints violated by optimal strategies are identified. Thus, those which effect future decisions are valued in the sense that they can be priced by comparing with and without analyses.

## Model Constraints

The urban water supply and distribution system model is bounded and operated by two major types of constraints: (1) water quality constraints; and (2) water flow constraints. The water quality constraints in effect place an upper limit on the quality flows diverted to each of the three use categories. It has been assumed that tertiary treatment would be necessary for any quantities of water recycled since public contact, occurring in all use categories, would forbid objectionable odors and potential health hazards resulting from the higher levels of BOD. In addition, it is unlikely that desalting could be feasibly accomplished if the suspended solids were limited only by primary and secondary wastewater treatment.

To begin, the quantity of water treated in the municipal raw water treatment system and its associated quality are defined,

$$X_5 = \sum_{j=1}^{4} X_j + (Q_r - X_7 - X_9) \quad \ldots \ldots \quad (65)$$

$$C_5 = \frac{\sum_{j=1}^{4} X_j C_j + (Q_r - X_7 - X_9)(C_{tr})}{D_d + X_6} \quad \ldots \ldots \quad (66)$$

in which:

$X_j$ = quantity of water from source j

$X_7$, & $X_9$ = quantities of water recycled directly to municipal and industrial uses

$Q_r$ = total quantity of water from reclaimed wastewater

$C_j$ = associated water quality (TDS) from respective sources

$C_{tr}$ = the TDS concentrations in the reuse water

$D_d$ = domestic demand

Then for each water use, an upper limit on TDS can be defined,

$$C_d - C_5 \geq 0 \quad \ldots \ldots \quad (67)$$

$$C_m - \left[\frac{X_6 C_5 + X_7 C_{tr}}{D_m + X_8}\right] \geq 0 \quad \ldots \ldots \quad (68)$$

$$C_i - \left[\frac{X_8 C_5 + X_9 C_{tr}}{D_i}\right] \geq 0 \quad \ldots \ldots \quad (69)$$

$$C_{ti} = \phi(C_d, C_m, C_i) \quad \ldots\ldots\ldots\ldots \quad (70)$$

where

$C_d$, $C_m$, $C_i$ = maximum allowable TDS for domestic, municipal, and industrial uses.

$C_{ti}$ = water quality of wastewater as a function of the water quality supplied to the demands.

$D_m$ and $D_i$ = municipal and industrial demands.

These constraints can be varied over an applicable range to determine the costs associated with delivering water to users of various quality.

The physical flow constraints become necessary not only to insure each demand is satisfied, but also to maintain feasible solutions. Because the model operates in an optimization format, it is necessary to provide for realistic solutions as the model minimizes costs. These constraints on the system can be written:

$$D_d + D_m + D_i = \sum_{j=1}^{4} X_i + Q_r \quad \ldots\ldots\ldots\ldots \quad (71)$$

$$D_m + X_8 - X_6 - X_7 = 0 \quad \ldots\ldots\ldots\ldots \quad (72)$$

$$D_i - X_8 - X_9 = 0 \quad \ldots\ldots\ldots\ldots \quad (73)$$

and,

$$Q_i = \phi(D_d, D_m, D_i) \quad \ldots\ldots\ldots\ldots \quad (74)$$

where $Q_i$ is the discharge of urban effluent, mgd.

## Objective Function

Employing the minimum cost criterion described in Chapter 2, and the cost functions described in the previous paragraphs, the objective function can be formulated. The objective function consists of three basic segments; (1) cost of water supplies available at the raw water treatment intake; (2) the costs of raw water treatment; and (3) the costs of recycling wastewater directly to the municipal and urban demands.

If $P_1$ is defined as the water supply cost, then

$$P_1 = \sum_{j=1}^{4} c_j X_j + (Q_r - X_7 - X_9)c_r \quad . \quad . \quad . \quad . \quad . \quad . \qquad (75)$$

in which $c_j$ is the unit costs of the $j^{th}$ water source, $X_j$ is the quantity selected from the $j^{th}$ source, and $c_r$ is the costs of reused water, derived from the regression with values of TDS in the flows. The polynomial expression for $c_r$ will be in the following form,

$$c_r = A_1 + A_2 C_{tr} + A_3 C_{tr}^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (76)$$

where $A_i$ are the regression coefficients. The variable $C_{tr}$ is the coordinating link between the urban wastewater reclamation system and the urban water supply and distribution system.

The Illinois State Water Survey (1968) indicated that in terms of 1964 dollar value, the capital construction costs for raw water treatment facilities, including screening, flocculation, clarification, rapid sand filtration, and chlorination, could be expressed as:

$$Y = 0.323\ Z^{0.65} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (77)$$

In addition, this source also showed the operation and maintenance costs could be determined by:

$$y = 1.21\ Z^{-0.35} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (78)$$

where y is the costs of the treated water in $/1000 gal and Z is the capacity of the treatment facilities in mgd. If these costs are evaluated at their present-worth, added to the interest costs, and extended in a uniform series over a design life of 30 years with a discount rate of 5%, then the total annual costs, in terms of 1964 dollars, can be written as,

$$P_2 = 200\ Z^{0.74} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (79)$$

where $P_2$ represents the total annual costs in dollars per million gallons treated.

The final component of the objective function is the costs of the recycled water which is sent directly to municipal and industrial demands. From the preceding definitions, $P_3$ can be representative of the costs and written as,

$$P_3 = (X_7 + X_9)c_r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (80)$$

or more completely:

$$P_3 = (X_7 + X_9)(A_1 + A_2C_{tr} + A_3C_{tr}^2) \quad . \quad . \quad . \quad . \quad . \quad (81)$$

The complete objective function can now be expressed:

$$y = \min_{i} \sum_{i=1}^{3} P_i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (82)$$

These previous functions do not include several of the costs encountered in supplying water to urban demands. For example, the costs of the distribution system, pumping, and storage are not included since they are assumed to be common to all aspects of the model. In addition, the functions were developed on the basis of differing dollar values which can be corrected by applying an adjustment factor taken from published cost indexes.

A further discussion of the Urban Water System Model as it actually operates has been included in Appendix A to aid the reader in understanding the model. In addition, Appendix B has been included to describe the water management characteristics of the Denver, Colorado metropolitan area upon which the model is to be applied.

# Chapter 6

# ANALYSIS OF URBAN WATER MANAGEMENT STRATEGIES

## Introduction

In the metropolitan setting, water management includes accounting of the supply, measuring the demand, and allocating efficiently according to political reality and economic ability (Flack, 1971). Accomplishing these objectives requires the continual improvement of the methods used to manage water under conditions of competition and scarcity.

This chapter presents the analysis obtained from the urban water management model which hopefully extends the knowledge concerning such water management policies. The results are divided into an examination of the essential system characteristics, evaluation of important institutional structures, and the future strategies suggested by expected conditions in the urban area. Although these conclusions apply only to Denver, it is hoped that they have been derived in a general enough manner to apply broadly to other arid urban areas as well.

## Characteristics of Urban Water Management Policies

The alternative water supplies for the area of Denver can be reasonably limited to interbasin transfers, stream flows, and reuse. Groundwater is generally omitted because of the unconfined nature of the stream-aquifer system and the consequent contamination of the groundwater supplies. The transfer of in-basin agricultural water rights is also an important water source, but is difficult to examine directly in the model. Consequently, it is necessary to evaluate the potential for this source indirectly in order to test its feasibility by varying the permissible levels of reuse. Since the system depletes approximately 50% of the inflows, allowing reuse of more than 50% of the interbasin transfers actually implies the reuse of some in-basin flows, which thus gives some indication of the costs or price of agricultural water right acquisition.

As water supplies become pressed to satisfy the needs, sources with poorer quality must be utilized. This is especially applicable in the case of recycled water, which becomes less expensive as its TDS levels are increased. As an indication of the effects of placing a priority on the quality of the flows within the urban distribution system, three distribution philosophies are explored:

(1) Alternative 1. Public attitudes, legal restrictions, or the physical structure of the distribution system, may require that wastewater be only recycled through existing raw water facilities. In this situation, where recycling directly to municipal and industrial demands is not possible, the reused flows are blended with the other

sources. As noted in previous sections, a limit on TDS in this case will be set at the highest level currently encountered, so that domestic supplies remain undegraded.

(2) Alternative 2a. Recycling individually to municipal and industrial demands may be permissible, but only if water quality levels are maintained at their current values. This policy may be inefficient from the standpoint of supplying individual demands with permissible water quality characteristics, but adds flexibility to the system.

(3) Alternative 2b. Probably the most economical long range distribution strategy for arid urban areas is to supply each need with water at the tolerable limits of TDS for each use. This practice allows the best water to be used for the most sensitive use and the poorest for the least sensitive. To accomplish this investigation, the TDS levels allowable for municipal uses is set at 800 mg/l while that for industrial is set at 500 mg/l. This alternative will serve as the basis for comparison between various combinations of alternatives.

Intuitively, if a more degraded water is supplied to the urban demands, the effluent quality will also reflect the change. Consequently, if recycling is employed, the

TDS levels in the area's effluent will increase. Since downstream water rights demand maintenance of quality as well as quantity, the effects of reuse are important considerations. An assumption has been made that these TDS effects are proportional to the changes in TDS supplied to the demands. In a limited search, little information on the Denver area was available to answer this question and the simplified approach appears necessary.

In this section, a thorough analysis of conditions as they existed in the early 1970's will be made to develop the basic characteristics of the model.

## Optimal Policy Spaces

When the institutional limitations on the model are temporarily ignored, optimal water management policies depend exclusively on the relative feasibility of the recycled water. This characteristic results from the assumption of linear cost functions representing the interbasin transfers and in-basin stream flow diversions. Since the average costs for recycled water depend on effluent BOD and TDS goals, the optimal water management decisions can therefore be expressed in terms of these variables. In this manner, water quality considerations are linked to the problems of water supply and distribution and thus an integration of both aspects of water management in an urban area is accomplished. When the decision for each point corresponding to a specified value of effluent BOD and TDS standard is evaluated, the policy space is defined and can be presented graphically. The term "policy space," whether optimal or not, is thus a representation of decisions which are shown as functions of important system parameters.

The optimal policy space for the first distribution scheme, Alternative 1, is shown in Figure 15. Under the assumptions of this alternative, the quantities of water diverted from each of the sources depends upon their supply costs. Since the in-basin stream flows are relatively inexpensive to deliver, all decisions would incorporate stream flows in their available quantities. As a result, the optimal strategies deal principally with the trade-off between interbasin transfers and recycled wastewater. The three areas delineated in Figure 15 represent: (1) zero reuse, (2) reuse limited by TDS concentrations, and (3) reuse limited by the constraint on the allowable percentage of interbasin transfers which are available for recycling.

In the space occupied by a zero reuse strategy most present day conditions are encountered, thereby indicating wastewater costs are yet higher than the costs of the other two sources. The "all or nothing" type strategies resulting from the linearity of these costs are demonstrated in the plot as illustrated by the sharp step-like nature of the boundaries between respective areas. A relatively large percentage of the plot represented by the zero reuse

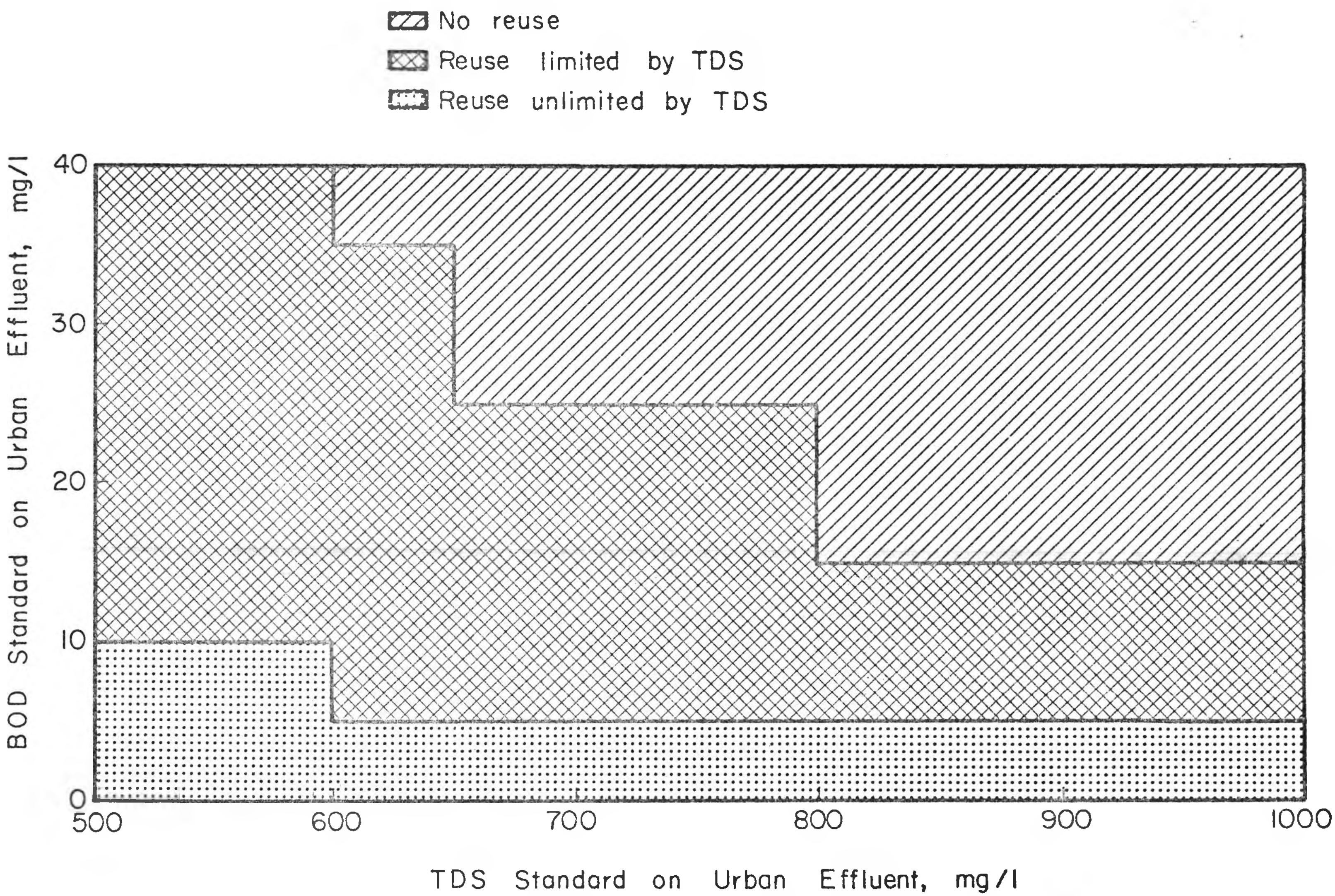


Figure 15. Optimal water management policy space of Alternative 1.

sector in this system of management supports many of the current plans being implemented in the Denver area.

The transistional area in Figure 15, representing conditions where the quantities of reuse used are dependent upon the quality of the flows, is applicable to situations of water quality management which are expected in the near future. The limitation on reuse in this sector of the plot results from the costs of reused water with low TDS concentrations not being competitive with the other sources. However, at the higher TDS concentrations, where the costs are competitive, the water quality constraint in the model protecting the quality for the domestic demands is activated and only a portion of the permissible reuse is utilized.

The final sector of the optimal policy space shown for Alternative 1 in Figure 15 is the condition when water quality standards on the urban outflow are sufficiently rigid to insure feasible recycling at any TDS level in the expected range. Recalling that reuse costs are the unit differences between the total costs without reuse and the total costs with a specified level of reuse, this sector illustrates the decrease in reuse costs as effluent quality levels are restricted.

The optimal strategies discussed for Alternative 1 were also generated for Alternatives 2a and 2b. It is probably worth noting, however, that unlike Alternative 1, these two remaining alternatives compare the feasibility of water sources at the point of demand delivery. In

Alternative 1, the sources were evaluated on the basis of supply cost, but in Alternatives 2a and 2b, the interbasin transfers and in-basin stream flows are increased in price by the costs of raw water treatment.

The optimal policy space for Alternative 2a, shown in Figure 16, illustrates the increased use of recycled water even when present quality criteria are met. The blending of recycled water directly with diversions from the other sources is sufficient to expand the region of unlimited reuse to include most of the space. There is a significant reduction in the transitional zone between Figure 15 and 16. The importance of being aware of this occurrance is that the optimal decisions are essentially ones of whether to reuse or not.

The distribution of the sectors in the policy space of Figure 16 suggest that if it is possible to recycle wastewater directly to municipal and industrial demands, plans should be rapidly made to do so because the boundaries of this area are very near present conditions. In this figure, the quality parameter most affecting the decision is the BOD standard on the urban effluent. This characteristic is markedly different from the results shown in Figure 15, which are about balanced between TDS and BOD effects. Because increased BOD removal efficiencies are expected sooner than requirements for TDS removals, the need for immediate planning is apparent.

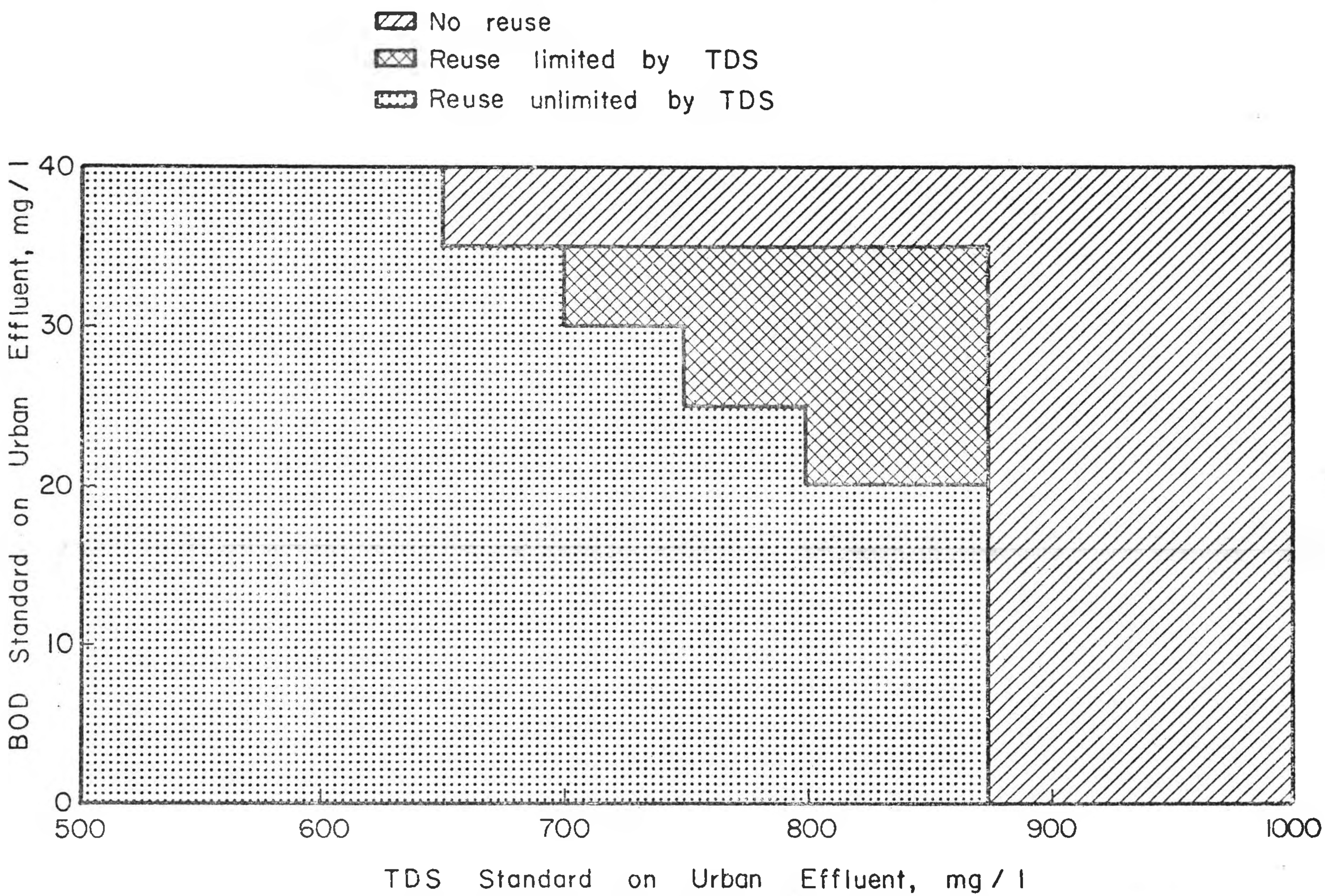


Figure 16. Optimal water management policy space for Alternative 2a.

The majority of the policy space occupied by the unlimited reuse sector suggests the need for separate distribution systems to serve both domestic as well as the municipal and industrial demands. The feasibility of constructing such dual systems as part of new developments, or as part of system rehabilitation, will be left to the water planner, but the costs of the alternatives will be shown later.

The final alternative, recycling to individual urban demands based on relaxed water quality goals (Alternative 2b), is shown in Figure 17. Again, the limits of the distribution system should be expanded to a dual system. Because of the zoning in most cities, a separate system for industrial reuse may not be too difficult to achieve.

The examination of the preceding policy space charts illustrated the feasibility for recycling wastewater in the metropolitan environment. The expected requirements for more refined wastewater treatment before releasing these flows to downstream users is obviously in favor of the recycling concept.

## Water Supply Costs

In addition to supply and treatment costs, the expenditures and investments necessary to supply a city with the water resources it needs included distribution networks, storage and pumping facilities, and metering and control structures. In the analysis presented herein, these

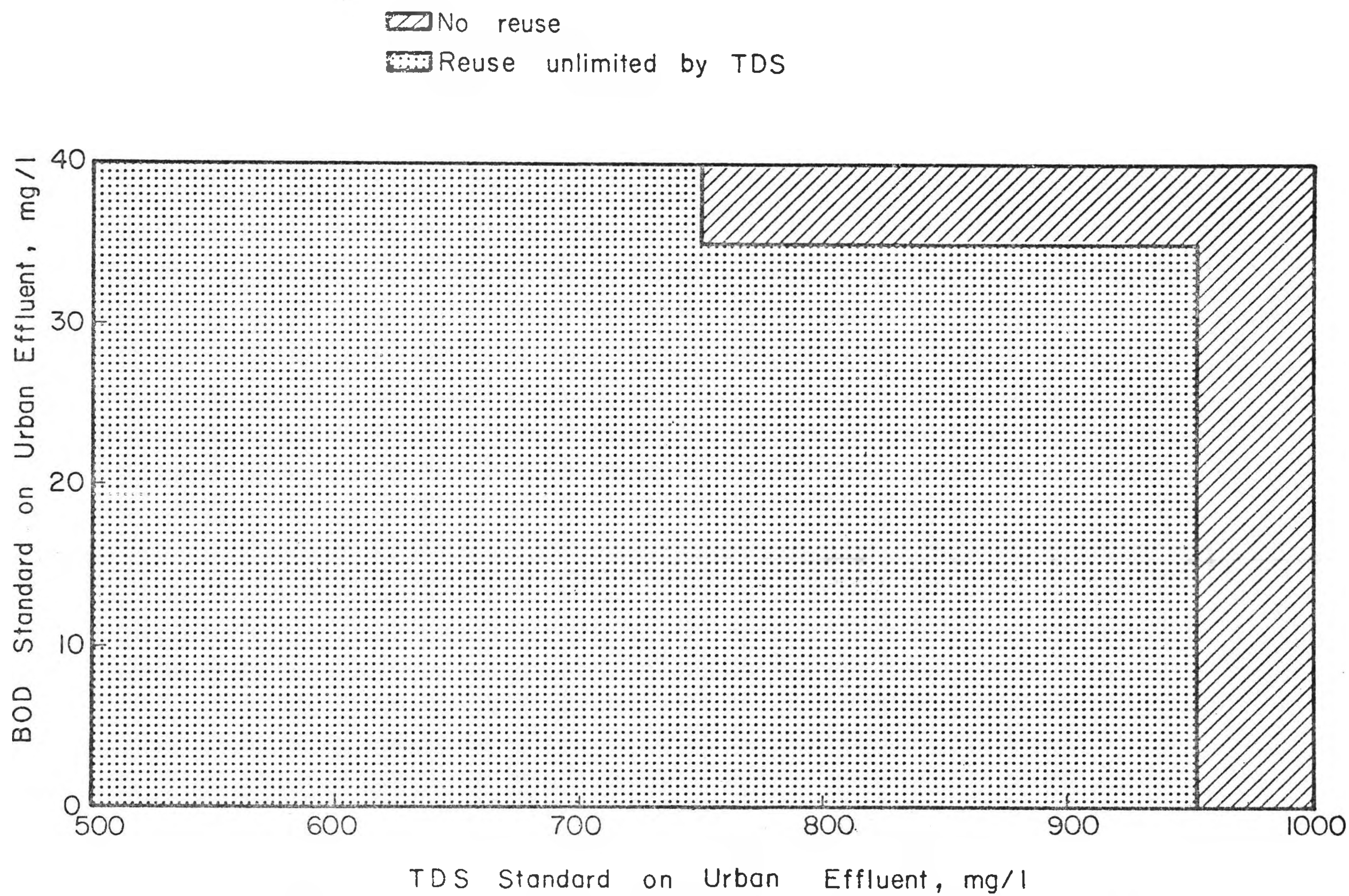

Figure 17. Optimal water management policy space for Alternative 2b.

additional costs have been omitted because of their occurrence in all planning alternatives. A comparison of model data with annual reports for the Denver area indicate the model analysis accounts for approximately 70-75% of the total annual expenditures.

The urban water system model was designed to accept mixing or blending of water supplies at each individual urban demand. Furthermore, the same structure in the wastewater treatment and reclamation model discussed previously was implemented. The effect of this blending ability is to render these cost functions unpredictable in shape unless one or two of the model parameters are completely dominating. From the previous section, it was shown that three basic policies were involved in evaluating optimal water management strategies. In each segment of the policy space, the combinations of reuse quality and blending ratio may be numerous. Consequently, if a cost function traversed several of these decision policies, its form may well be irregular.

The effects of effluent standards on urban water management decisions have been demonstrated to be relatively important. As a consequence of this and in order to provide a consistant presentation, the relationships in this and following sections will be plotted against effluent quality standards. Because of the irregular nature of the cost functions to be presented in this section, which illustrate many facets of the models operation, it is helpful to detail the causes for these irregular functions. In addition, several of the analyses in the following sections exhibit the unexpected geometries shown in this section. So, rather than repeatedly explain such irregularities, care will be taken here to note the reasons for the behavior of the model results.

The annual water supply costs have been plotted in terms of 1970 dollars in Figures 18 and 19 for Alternatives 2a and 2b. In addition to the curves for the various BOD standards, points are also plotted for the case in which 100% of the interbasin transfers are recycled. The relevance of the differences between the cost functions for the two magnitudes of recycling (50% and 100%) will be shown in a later section.

There are three primary characteristics of the urban water system model shown in Figures 18 and 19. The first can be demonstrated by examining the curve representing an effluent BOD constraint of 35 mg/l (CBO = 35 mg/l). In Figure 18, for example, the points between an effluent TDS standard of 500 mg/l and 700 mg/l define a monotonically increasing curve with decreasing marginal costs. Referring once again to Figure 16, the optimal policy space for this alternative, it can be seen that the decisions in this range of the curve lay in and along the unlimited reuse sector. Under this policy then, the only varying model parameters are the effluent TDS constraints and thus the desalting plant, as well as the prepatory tertiary

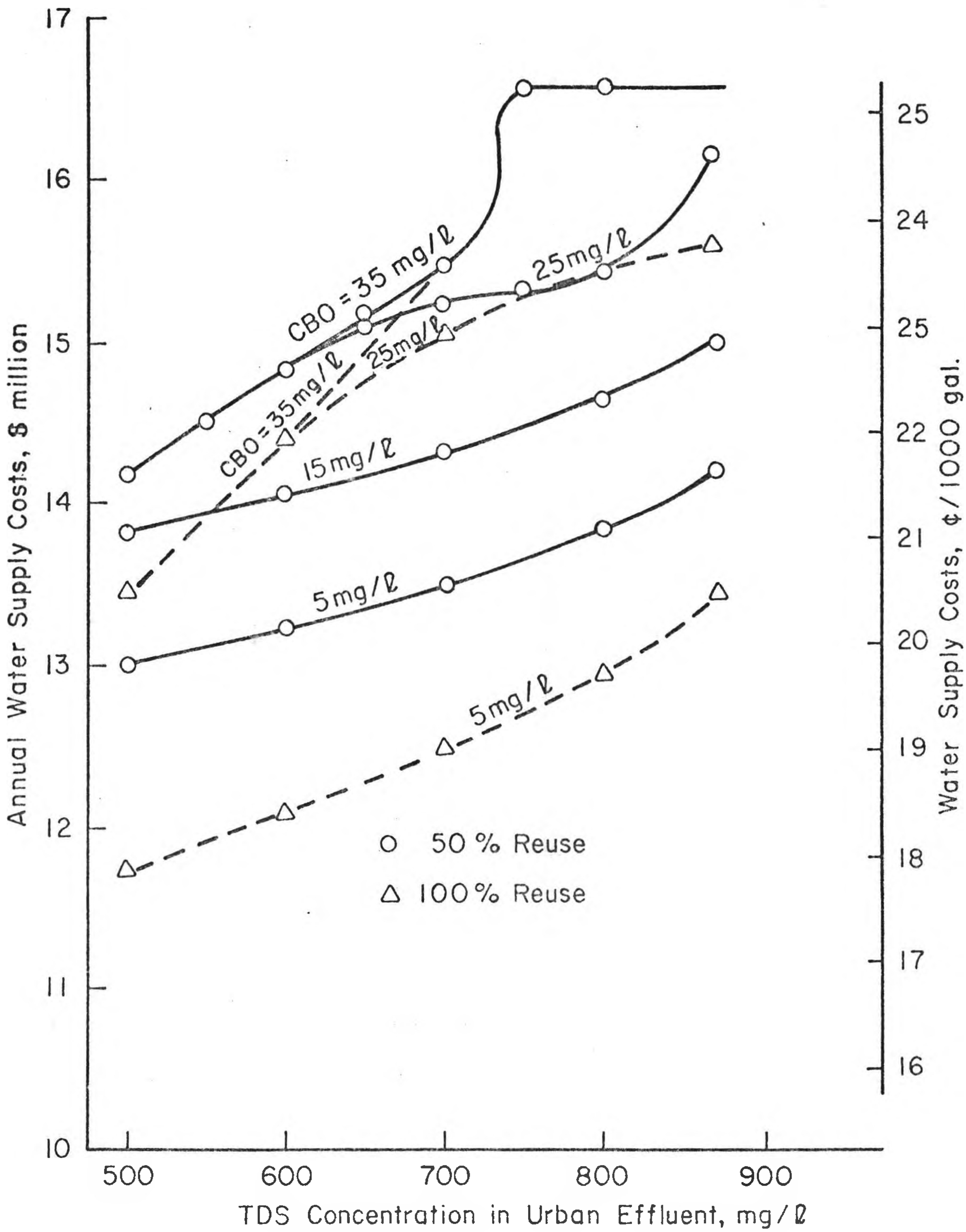


Figure 18. Annual water supply costs of distribution Alternative 2a.

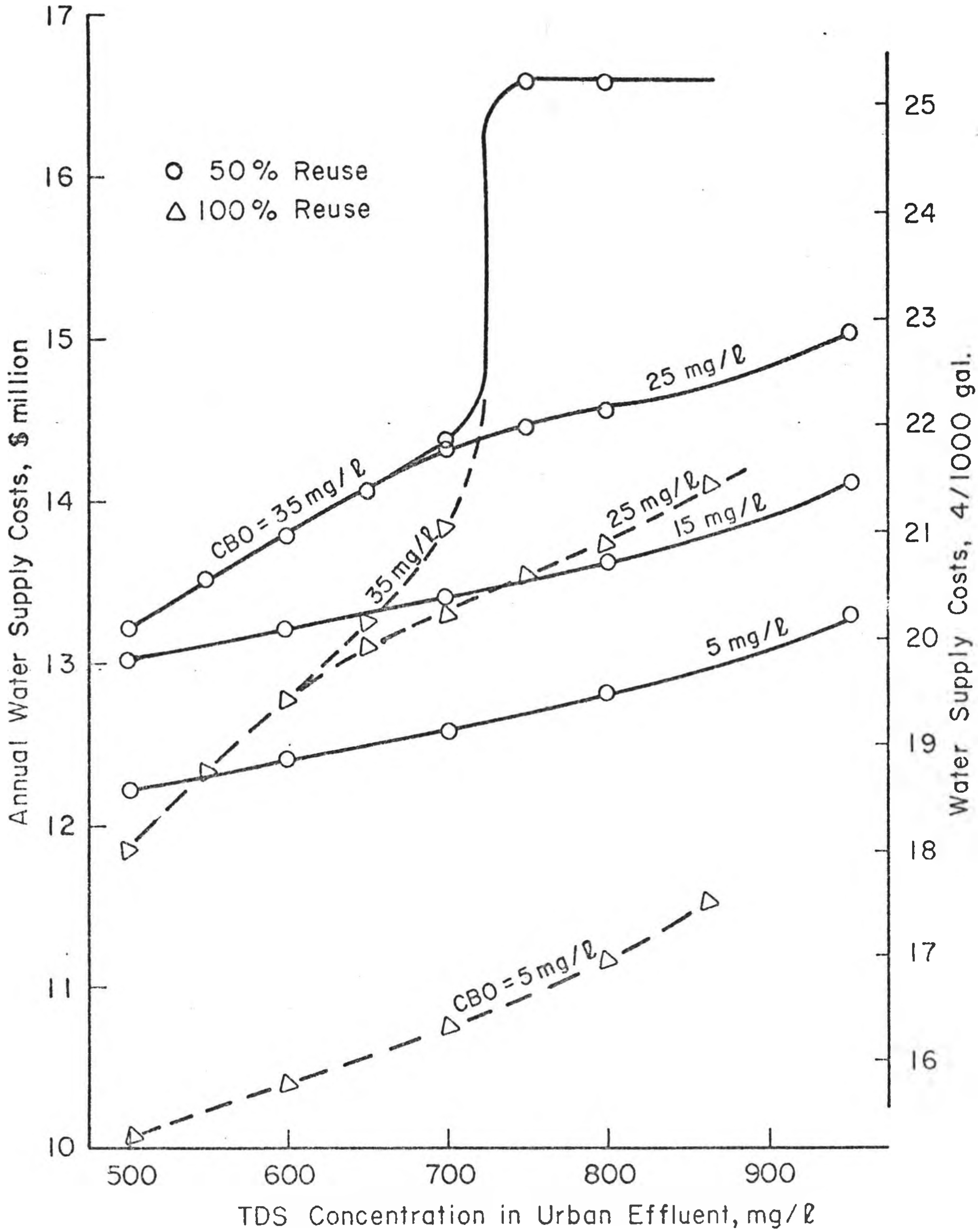


Figure 19. Annual water supply costs for distribution Alternative 2b.

treatment capacity. The cost functions depicted in Figure 18, therefore, not only illustrate the decreasing marginal cost characteristics of the wastewater treatment facilities, but also a reduction in the relative price advantage of the reused flows. At the interval between 700 and 750 mg/l on the abscissa, the strategy passes rapidly through first the limited reuse sector into the zero reuse sector. This transition is not clearly indicated in Figure 16 because the CBO = 35 mg/l points mark the boundary between the two sectors of the plot. The upward transition curve in Figure 18 at about 720-730 mg/l ends in a vertical segment which then is joined by a horizontal line to another curving transition. The linearities of the cost functions in the model create this geometry in the region, the "all or nothing" decisions, but should probably be a more uniform transition when better cost information can be utilized.

The second aspect of the model shown in Figures 18 and 19 is demonstrated in the 25 mg/l curves, which initially coincide with the 35 mg/l line. This equivalence for a segment of the curves rests with the fact that in this region, the mixing in the effluents necessary to achieve the reduced TDS levels causes the actual effluent BOD concentrations to be below the constraint. Examination of the computer printouts revealed that the concentration of TDS in the reuse was the same for both BOD limits. Consequently, in both of these instances the tertiary and desalting capacities were the same, thereby causing the water supply costs to also be the same. After this initial segment of coincidence, the 25 mg/l curves separate and maintain a function exhibiting the decreasing marginal costs of the treatment facilities. In this situation, however, a mix is achieved in the effluent which satisfies both quality constraints. And finally, the last segment of the curve turns upward, which reflects the change in reuse policies described in the previous paragraph.

Finally, for the BOD levels of 15 and 5 mg/l, the curves show no effect of either policy changes or waste-water treatment configurations. When this stability is reached, the relationships can be summarized for the various alternatives at selected levels of permissible re-use, as shown in Figure 20, in order to visually compare differences.

Another view of the water supply cost curves is presented in Figure 21 where the effluent TDS levels have been fixed at two levels, namely 550 and 750 mg/l. The two upper plots represent the distribution Alternative 2a in which the reuse to municipal and industrial demands are restricted by a 300 mg/l TDS constraint, while the two lower figures represent Alternative 2b, in which municipal flows are limited by a 800 mg/l constraint and industrial flows by a 500 mg/l restriction. The range of effluent BOD standards is plotted along the ordinate and the annual water supply costs along the abscissa.

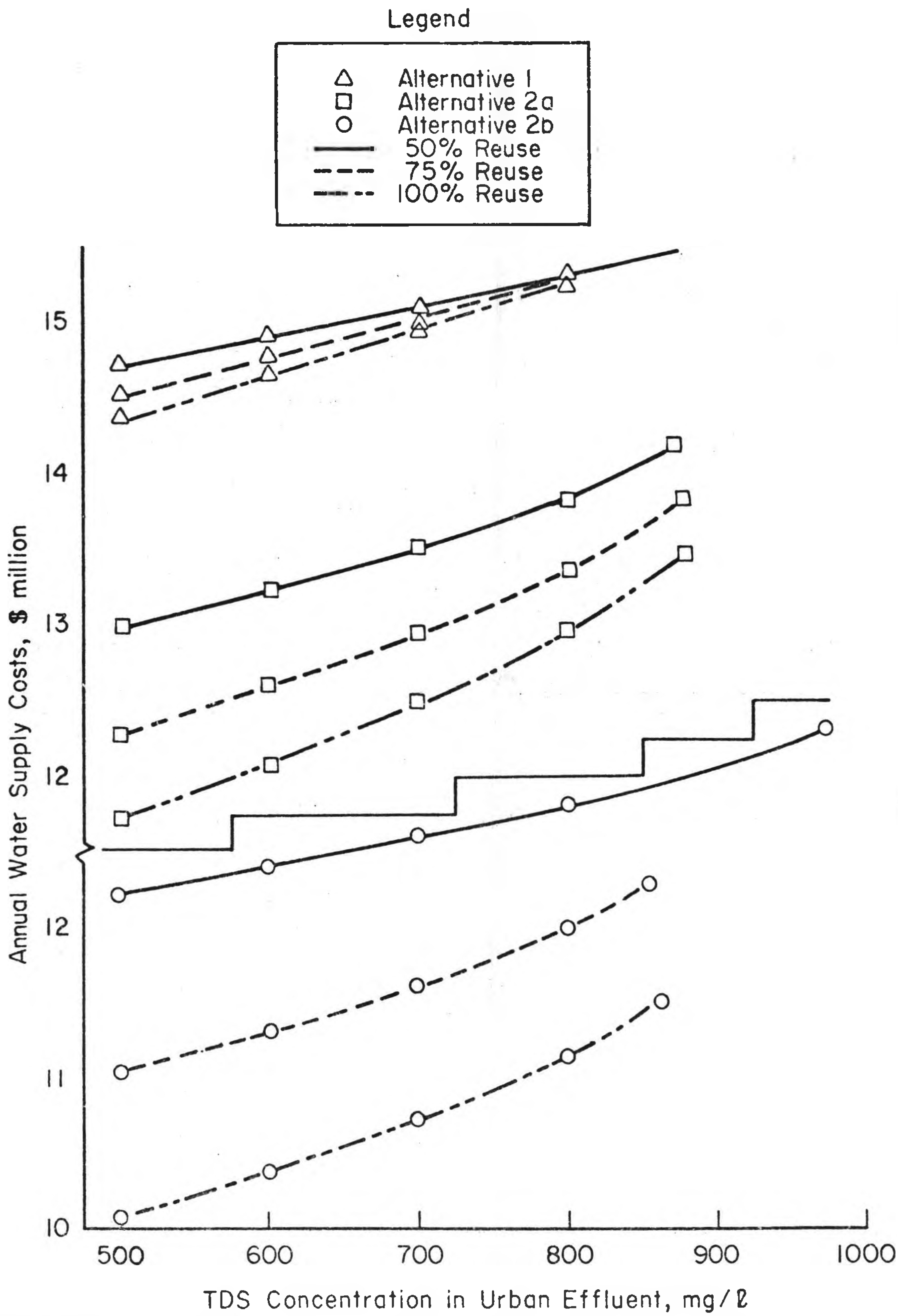


Figure 20. Annual water supply costs for the three distribution alternatives when the effluent BOD standard is fixed at 5 mg/l.

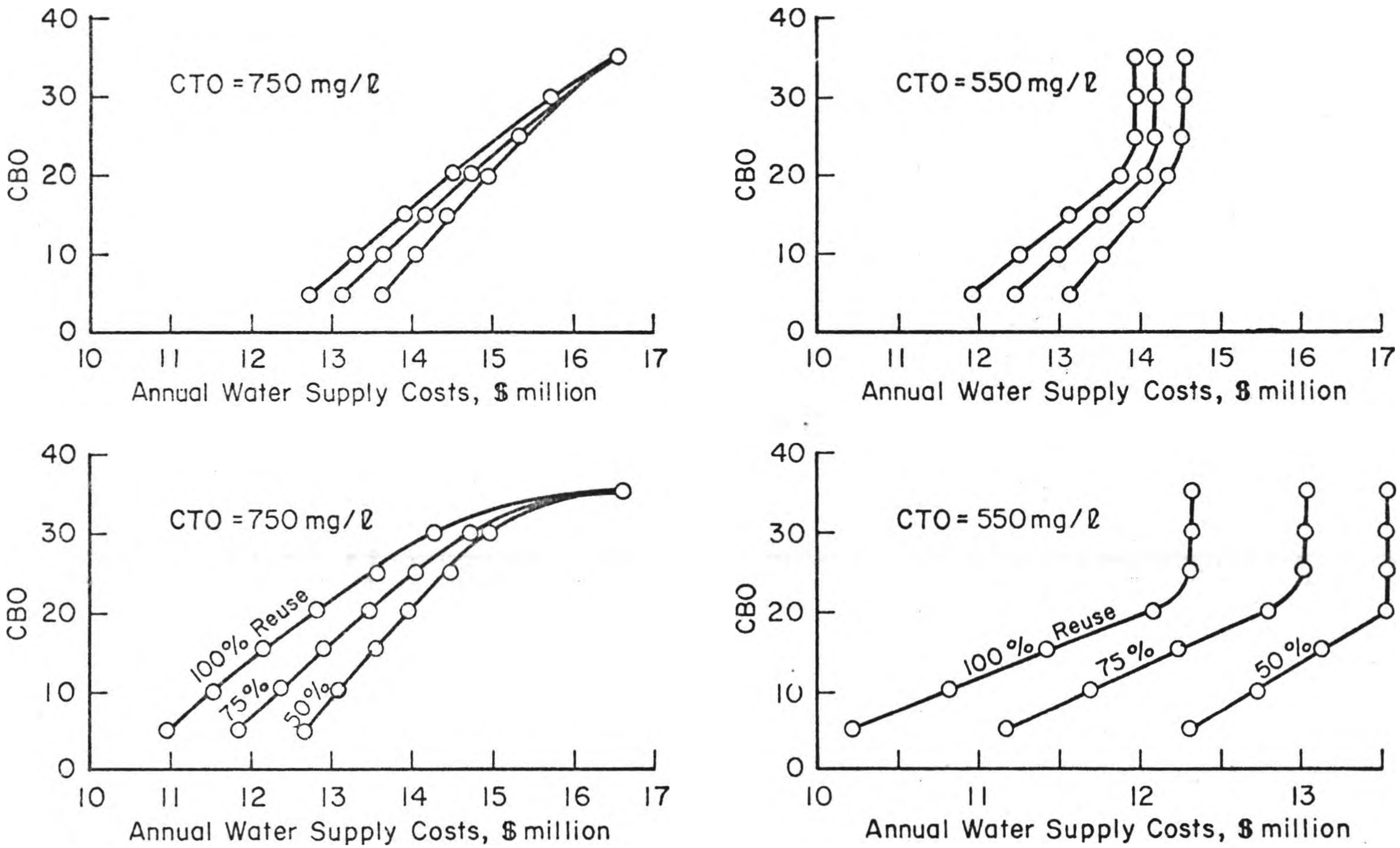


Figure 21. Annual water supply costs for distribution Alternatives 2a (upper curves) and 2b (lower curves) as functions of effluent BOD standards.

Drastically different relationships are shown in Figure 21 for the 750 mg/l and the 550 mg/l TDS levels. Again, the blending of flows in both the water supply and the wastewater treatment models is responsible. At a TDS standard of 750 mg/l on the urban effluent, the curves are smooth unimodal functions culminating at the single value noted previously as the zero reuse sector of the policy space. Considering first the 750 mg/l plots, the desalting plant capacity is fixed and thus the increase in the effluent BOD restrictions reduces only the capacity of the tertiary plant. If this is the case, it may be tempting to question why the costs do not actually decrease rather than increase. It is necessary to remember that as urban effluent water quality standards are relaxed, the costs of reused water increase resulting in less flows being reused. Thus, as recycled water becomes less competitive with the other water sources, the savings are also reduced. Eventually, no wastewater is reused and the optimal policy is at current conditions, as on the upper points of Figure 21.

In the curves representing the effluent TDS standard of 550 mg/l, the cost functions are independent of effluent BOD limits until about 20-25 mg/l, then the cost functions are similar to those for the 750 mg/l condition. The explanation for the occurrence of curves of this shape is the same as for the equivalent sectors in Figures 18 and 19, i.e., the desalting requirements are exclusively dictating the capacities of the systems. Until the BOD limits reach the 20-25 mg/l range, the TDS requirements dictate that more water must be subjected to tertiary treatment in order for desalting to be accomplished than is necessary to meet the BOD constraints. Consequently, no blending is possible and the curves are unaffected by BOD changes. When the BOD limits are lower than 20-25 mg/l, blending is possible and both TDS and BOD constraints are tightened.

## Total System Costs

Up to this point, the emphasis has centered on optimizing the urban water supply costs. Since recycling wastewater is one alternative source of water, the operation of the water supply and distribution model is dependent upon the corresponding operations of the wastewater treatment and reclamation model. The primary conclusion which has been substantiated is that as water quality standards on urban effluents become more stringent, the feasibility of adding enough capacity to the treatment elements and recycling some of the wastewater flow is enhanced. In the previous analysis, the savings to the supply agencies are shown to be substantial when recycling is implemented under optimum strategies. The question that may very well be asked is, "Why not impose extremely rigid water quality controls on ones' effluent in order to more cheaply supply the urban demands?" The answer lies in the observation that although the unit costs of reused water are shown to decrease, the total expenditure necessary to achieve more refined pollutant removal is increase increased.

In order to evaluate the effects of different water and wastewater management schemes, the total annual system costs were computed. As an illustration of these costs, the total annual system costs were plotted for Alternative 2a. This plot, shown in Figure 22, again indicates instability of the decision at the higher levels of effluent quality. For effluent BOD standards, CBO, of 25 and 35 mg/l, the decisions being made are whether or not to use any recycled wastewater, where the upper curves represent decisions as to the magnitude of the reuse and its associated quality. Included in Figure 22 are 75% and 100% reuse of interbasin transfers for the CBO values of 5 and 35 mg/l. The relative differences at 35 mg/l are small in comparison with the 5 mg/l differences, which is of interest in evaluating the value of agricultural water rights to the urban user as will be noted later.

From Figure 22, it is apparent that the savings to the water supply system by recycling wastewater are more than compensated for by the increased treatment costs when viewing the system as a whole. As these water quality goals are strengthened, the initial cost increases will be significant. For example, the total annual cost change will be from about \$24.3 million at CBO = 35 mg/l, to \$26.1 million

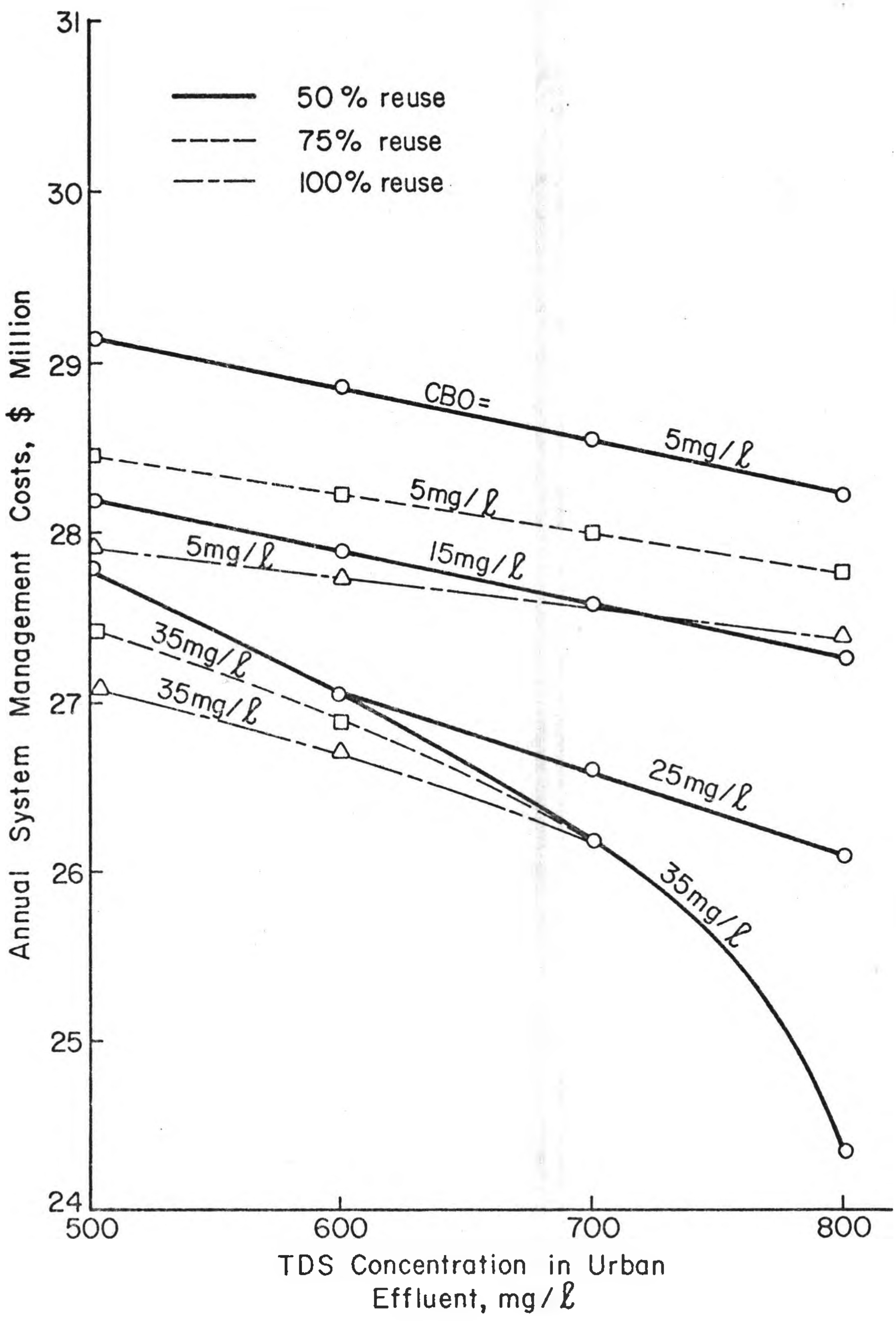


Figure 22. Annual system costs for distribution Alternative 2a.

at 25 mg/l, \$27.7 million at 15 mg/l, and \$28.2 million at 5 mg/l for an effluent TDS level of 800 mg/l. This is about a 15% increase which can be expected in the near future. If TDS standards as well as the BOD limits are enforced, the total cost increases will be approximately \$5.0 million annually or about a 20% increase.

Another important characteristic of the total cost function is illustrated in Figure 23. When the effluent BOD standard is fixed at 15 mg/l and the effluent TDS levels are allowed to range over the values for each of the three distribution alternatives, the effects of different levels of permissible reuse are shown to vary widely. For Alternative 2b the difference in total annual costs amounts to about 1.8 - 2.0 million dollars between the 50% and 100% reuse levels. Whereas the difference is only about \$0.5 - 1.0 million annually in the case of Alternative 2a, and almost no difference in the relationships for Alternative 1.

## Effects of Reuse on Water Quality

It would seem justifiable to state that if an increase in the concentration of TDS occurred in Denver's raw water supplies, the TDS concentration in the wastewater would in some manner reflect such an increase. To precisely predict the effect of water quality fluctuations on effluent quality characteristics would necessitate detailed examination of the water use sectors in the metropolitan area.

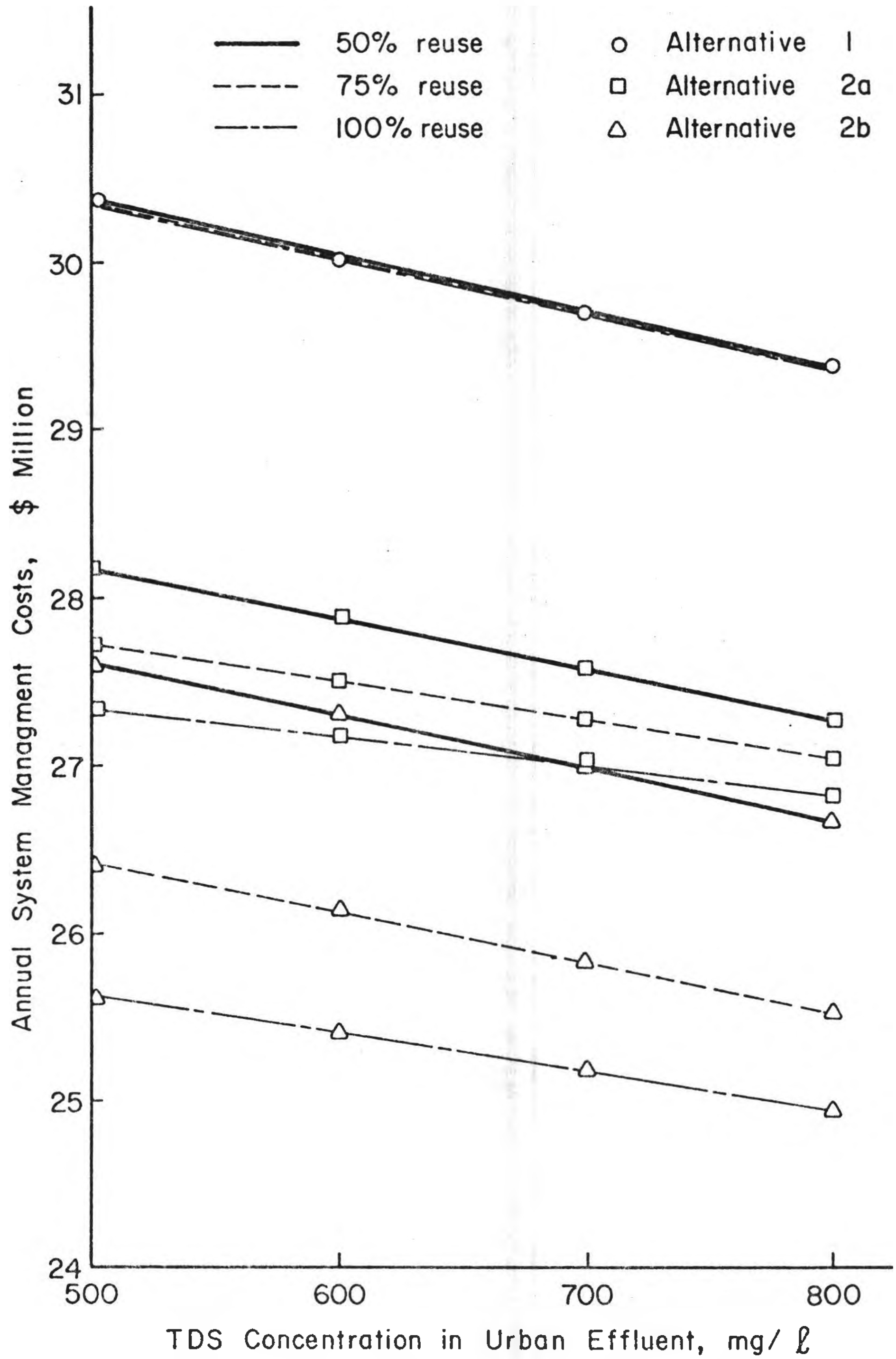


Figure 23. Annual system costs for the various distribution alternatives at an effluent BOD level of 15 mg/l.

Furthermore, the numerous sizes and composition of American cities would dictate that initial investigations would be primarily applicable only to the locale of the study. Since no such analysis has been made in the Denver area from which applications could be made in this analysis, it was assumed that influent quality changes would be reflected proportionately in the effluent. Some biasing was introduced in the assumption that only a small fraction of the flows supplied to municipal demands returns to the sewage collection system.

A simple water and salt balance on an urban area conclusively demonstrates that a "pickup" occurs in the system. Banks, et al (1971) suggests such a pickup amounts to an average increase in TDS concentrations, above the consumption concentrating effects, of about 300-400 mg/l for one cycle of use. In the Denver area, these pickup effects are on the order of 500-600 mg/l, but where such salt loads are acquired have not been delineated sufficienty to justify a more sophisticated analysis here.

The effects of recycling on TDS levels in wastewaters from the Denver metropolitan area is an important consideration. For the situation where recycling is accomplished through the existing raw water facilities, the exhibited water quality effects are governed by the constraints on domestic TDS levels. The results of this analysis indicates that the TDS concentration in the effluent flows could be expected to increase by about 200 mg/l when reuse is introduced. This increase is the largest encountered in the analysis and is due primarily to the fact that water of poorer quality cannot be directed to demands with high consumption ratios.

For the case of Alternative 2a, the TDS increases amounted to 70 mg/l. In this situation, most of the recycled water was diverted to the municipal demand where the return flow percentage is quite small and the effects on the wastewater were minimal. In addition, the maintenance of the domestic quality constraint (300 mg/l) limited the level of TDS in the recycled water to about 600 mg/l which further diminished the effects in the return flows.

Finally, the situation where recycling is accomplished according to Alternative 2b results in concentration increases of about 150 mg/l. The water quality constraints on both municipal and domestic demands became tight as the reuse quality approached 950 mg/l, indicating also that desalting was not necessary for the flows. This may be noted as the cost difference between the two latter recycling alternatives.

Because of the water quality constraints being tight in nearly all conditions of reuse, future policies and variable levels of reuse do not affect these results. Recycling does indeed affect downstream water quality and needs to be further evaluated.

## Evaluation of Water Management Institutions

Water management institutions comprise the vast and complicated array of legal, social, political, and economic structures invented to accomplish equitable allocation of water resources. As the foundation of management practices, these factors require periodic and detailed scrutiny in order for proper modifications to be made which reflect the evolving requirements for efficient water utilization. Unfortunately, little or no modification in the administrative apparatus has been successfully completed until problems approach crisis proportions.

The nature of institutional restrictions have quite often been evaluated too late. Consequently, a useful analysis which could be performed by integrating urban water supply with its counterpart, wastewater treatment systems, in such a manner as to optimize the complete system is to examine the institutional restrictions violated by such a derived policy. This investigation yields two important results:

(1) Those constraints most affecting the implementation of optimal strategies are identified; and

(2) The cost or value of the restriction is determined.

As a result of identifying institutional constraints in this manner, the decision maker and the public are provided with information that can be used to rationally select or recommend changes which would achieve more efficient water management in the urban setting.

In this section, several of the more important institutional questions have been selected for analysis. First, in order to assess the effects of increasingly stringent requirements for wastewater treatment capabilities, the costs of these future plans in the Denver metropolitan area are evaluated. Secondly, the power of the public to direct water management policies through approval or disapproval of funding is evaluated and various non-optimal decisions are compared. Third, the value of agricultural water rights are determined so the feasibility of considering this water source for future supplies can be determined. Fourth, the costs associated with maintenance of downstream TDS levels are calculated to quantify the often overlooked aspect of water rights upon water quality. And finally, a discussion of institutional consolidation is presented to stress the need for regional planning and service coordination.

Costs of Water Quality Controls

The impact of the Denver metropolitan area on the quality of flows in the lower reaches of the South Platte River basin would be over-whelming if measures were not taken to alleviate the burden on the stream flows. With the expanding needs for recreation and the like, it will be necessary for wastewater treatment to become more and more efficient. As a means of demonstrating the added costs derived from these policies, the difference between present costs and costs associated with various levels of pollution abatement were computed and plotted in Figure 26. Included in this illustration are the costs of those added treatment requirements when no coordination is attempted between water supply and wastewater treatment agencies. With no attempt to optimize the total system, the added costs (occurring mainly in wastewater treatment facilities) are substantial. From Figure 24, it can be observed that the effect of varying TDS levels is much more significant at the higher levels of BOD. For example, the slope of the 35 mg/l line in both cases is more negative than for the 5 or 15 mg/l lines. For the case where no optimization occurs, the lines (with the exception of the 35 mg/l line) level out above 800 mg/l. This characteristic has been assumed since the TDS levels in Denver's effluents do not normally exceed 800 mg/l. Consequently, the higher permissible TDS levels actually represent a loosening of any TDS constraints and is not reflected in the system costs. The radical departure of the 35 mg/l line is due again to the common policy among either alternative. Therefore, the added costs at present conditions are equal to zero as would be expected.

The savings resulting from the optimization can now be seen. If the differences between corresponding curves in Figure 24 are evaluated, as had been done in Figure 25,

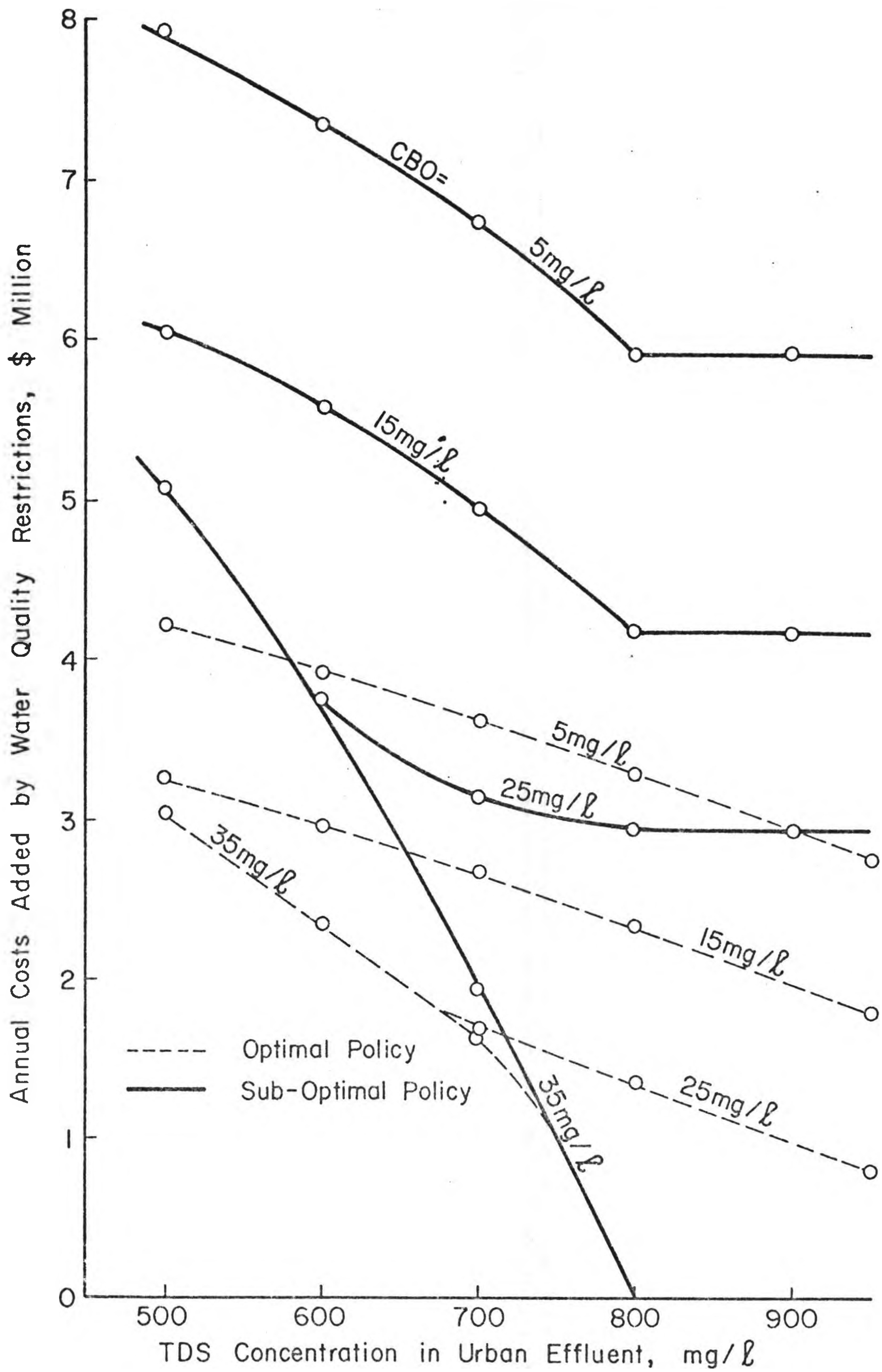


Figure 24. Annual system costs added by more rigid effluent quality standards under Alternative 2b.

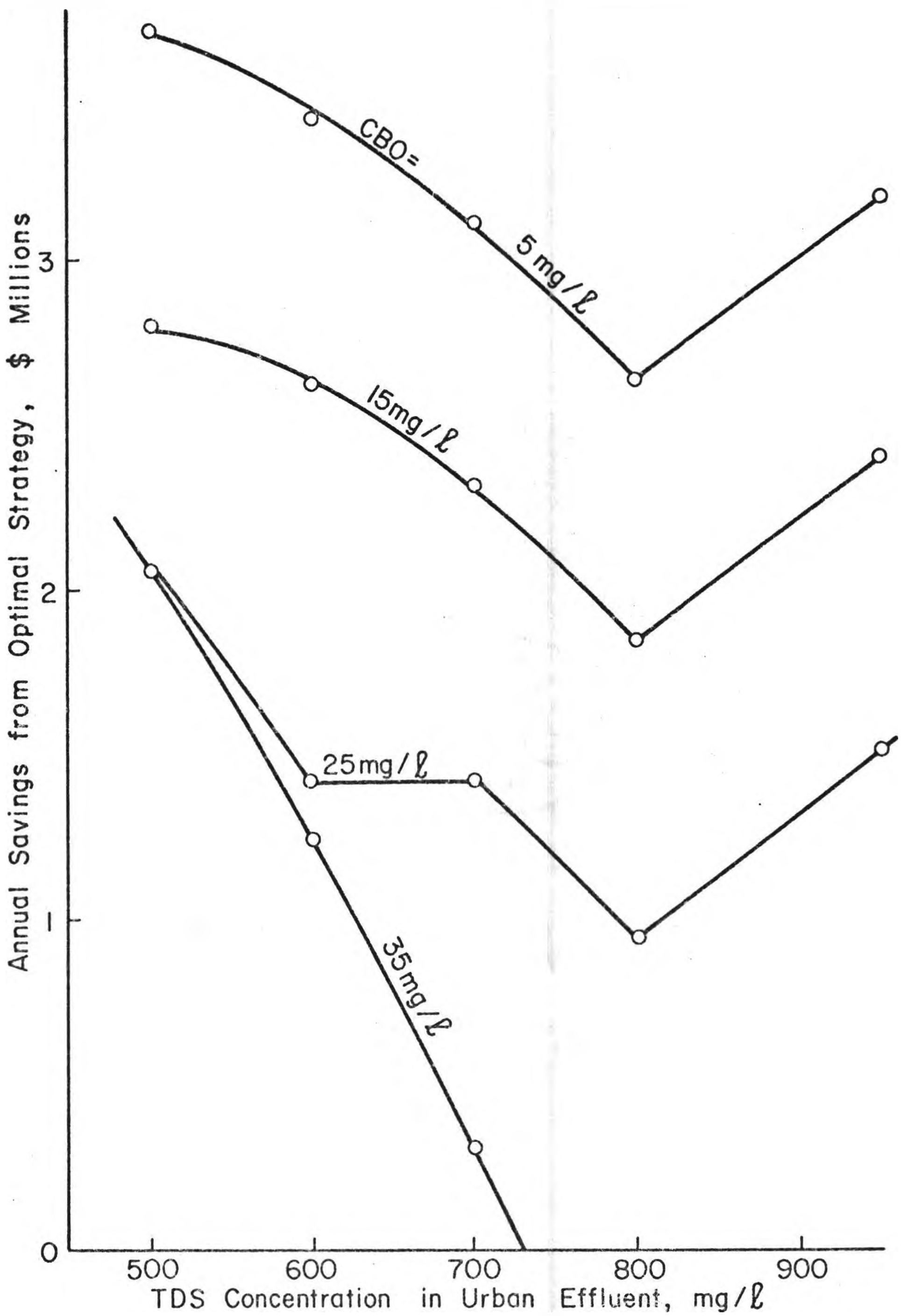


Figure 25. Annual system savings by optimal management policies as effluent quality standards are raised under Alternative 2b.

it is apparent that the need to implement optimal policies for urban water management is paramount. At an effluent BOD standard of 5 mg/l, for example, and no limit on effluent TDS, the system savings amounts to more than 3 million dollars per year, or about 5-10% of the annual system costs.

Even in a metropolitan area as large as Denver with large expenditures for water management, the costs of ignoring system coordination are significant. As water quality goals become more stringent (tighten), as they will in the near future, optimizing urban water management strategies will become more necessary.

Costs of Public Decisions and Attitudes

Public attitudes towards water management alternatives are undoubtedly the most important factor in future actions. In this regard, public attitudes are in reality an institutional force which, by controlling the funding for various projects, exercises the final decisions regarding feasibility. One of the strengths in this system of government is the decisions are made on the preference of the voters based on conclusions drawn from assimilated information. However, the source and intensity of information is critical to the voters choice.

Another problem is that some decisions can be made which are really detrimental in nature because of the lack of information available to the public. For example, decisions can be based on cultural connotations, like the thoughts of drinking sewer water. In order to quantify some of the effects of public disapproval of optimal policies, the model data was analyzed in the same fashion previously described. The results of several of these analysis follows.

From most of the previous analysis, the optimal water management strategies have stressed recycling as being extremely important because it promotes optimal coordination of urban water supply and urban wastewater policies. A comparison was made between the water supply costs as they currently exist and those under the strategy of recycling to municipal and industrial demands suggested by Alternative 2b. The results, shown in Figure 26, point out the high costs of not employing the optimal strategy. The curve representing 35 mg/l BOD shows the transition between the present and the optimal policies and indicates little difference between alternatives under present restrictions. At the lower concentrations of BOD in the urban effluent, the cost became substantial. For example, at a BOD level of 25 mg/l, the cost difference ranges between 2-3 million dollars annually. Comparison with the total water delivery system costs presented by the Board of Water Commissioners (1971) show this is about a 10% savings. If a comparison is made on the basis of water supply and treatment, omitting distribution system costs and the like which are common to the alternatives, this difference is approximately

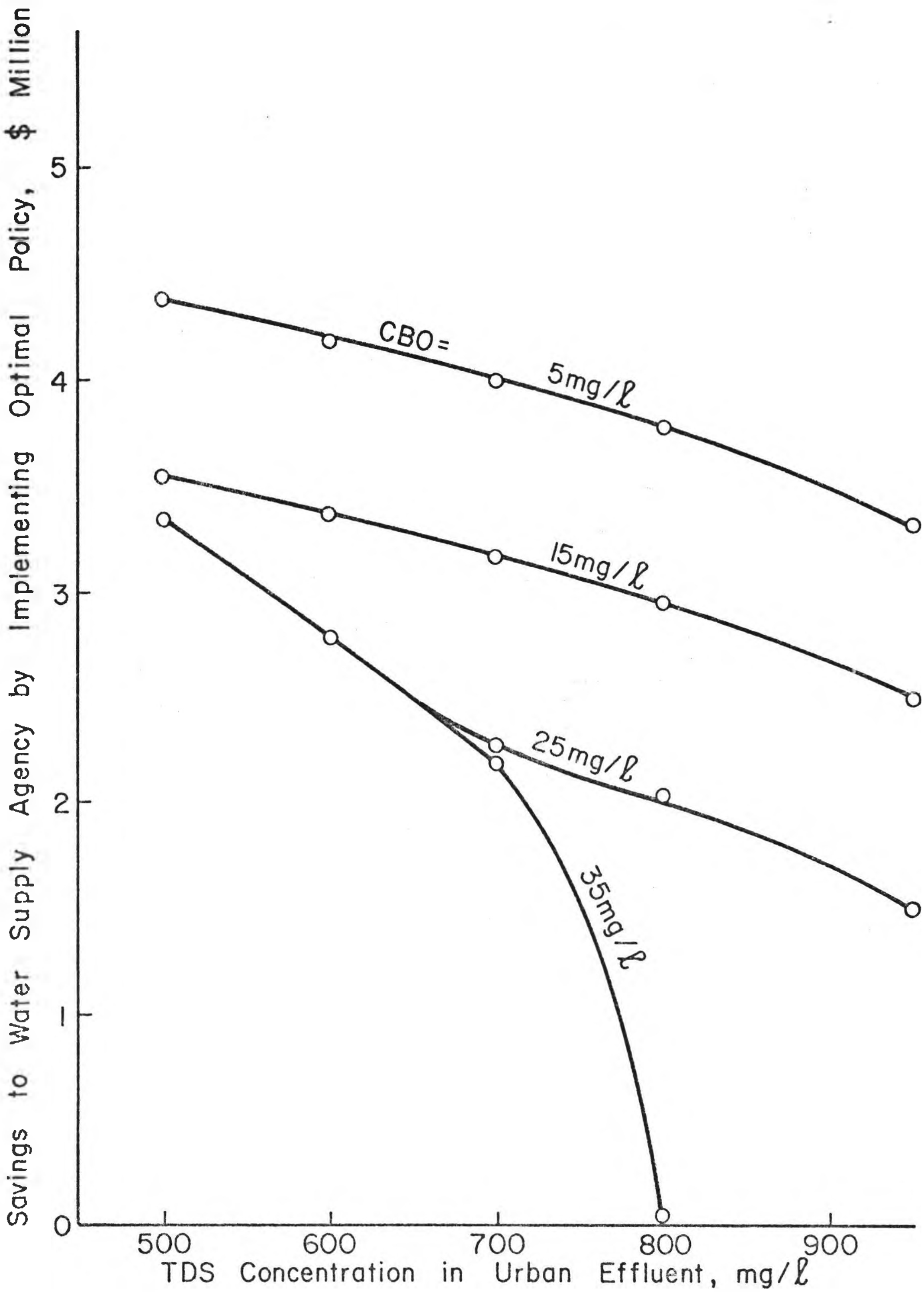


Figure 26. Savings to water supply agencies by implementing Alternative 2b.

15%. If the 5 mg/l curve is examined, a policy which can be expected quite soon, the differences amount to approximately 20% of the total annual water delivery system costs.

These costs can also be presented in terms of the expected savings in ¢/1000 gal, as shown in Figure 27, in order to present these results in typical urban water assessments. In 1971, the Board of Water Commissioners (1971) listed an aggregate water supply cost of about 22.6 million dollars for the 62 billion gallons consumed. This fugure thus represents a unit cost of about 36¢/1000 gal. Examination of Figure 27 reveals the savings in water rates is thus about 5-10% of the price listed above, but 15-20% of the actual supply and treatment costs. It should be noted, however, that since the recycled flows are being diverted to the municipal and industrial user exclusively, the price changes are much more likely to be reflected in these sectors. Consequently, the cost figures were derived for these sectors and the differences again calculated and presented in Figure 28. When computed in this manner, the savings are substantial, amounting to as much as 20%.

Before concluding this particular analysis it may be worthwhile to point out two more savings that can be achieved in an urban area. The analysis thus far has dealt with the policy of recycling under relaxed TDS constraints indicated by Alternative 2b. A similar analysis

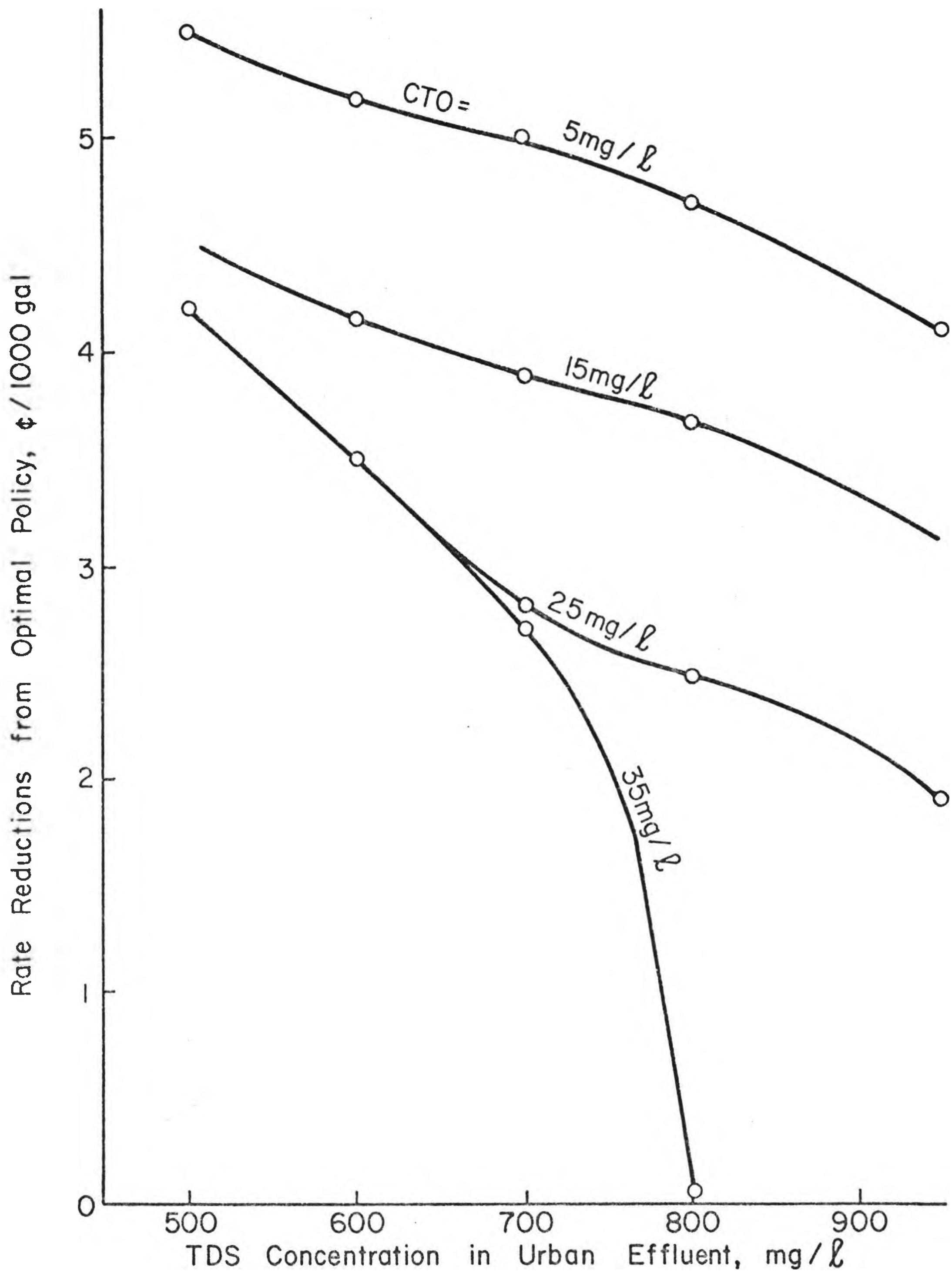


Figure 27. Possible rate reductions to urban water users by implementing Alternative 2b.

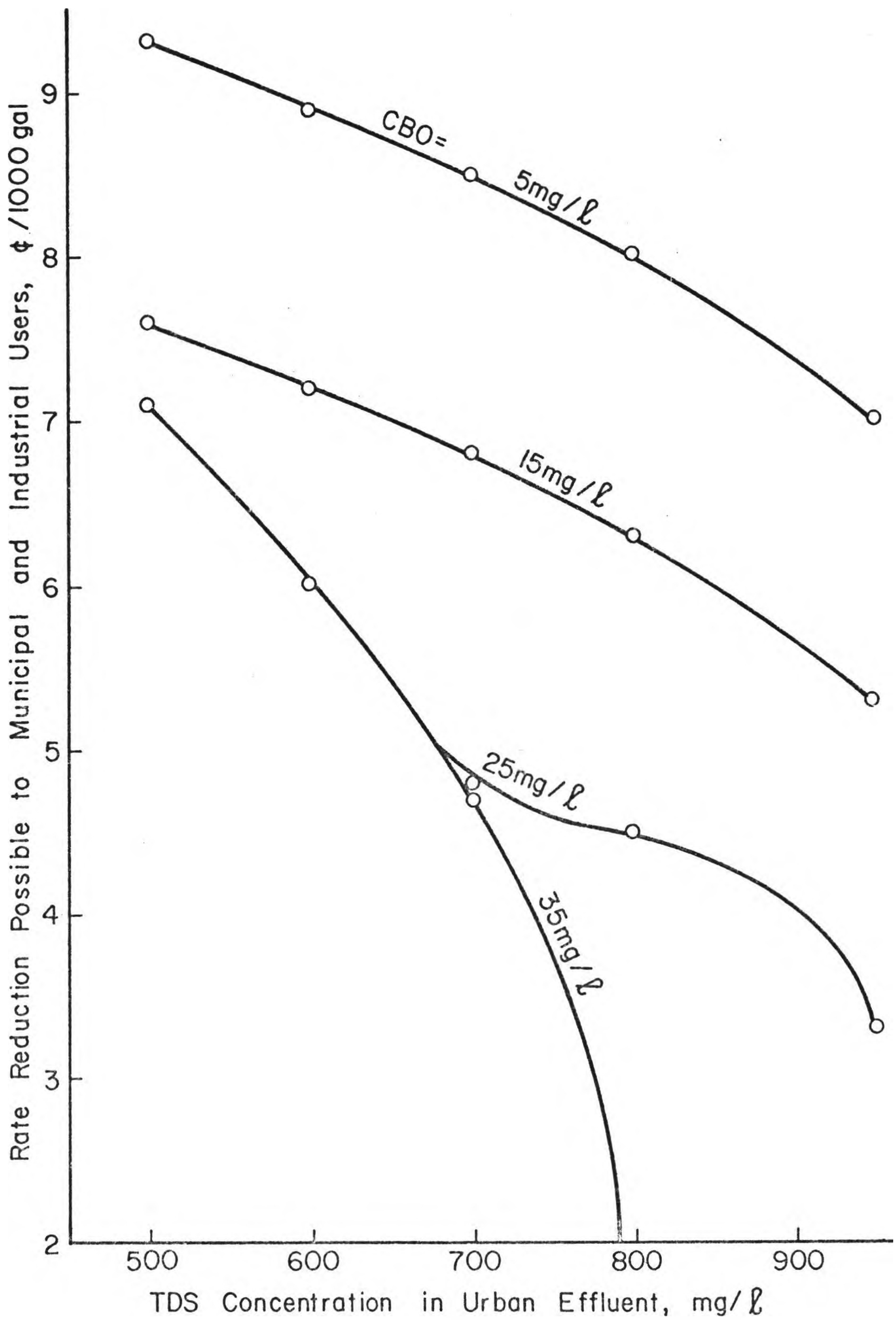


Figure 28. Possible rate reduction to municipal and industrial water users by implementing optimal strategies.

can be made for the costs associated with the other two alternatives, 1 and 2a.

First, a comparison of rates between the Alternative 2a policy and Alternative 2b is shown in Figure 29. At first glance, the array of curve shapes in this plot are confusing. However, keeping in mind that these curves reflect differences between curves with widely varying slopes the question may be somewhat alleviated. The significance of this plot is that it indicates the costs associated with the public attitudes regarding quality of recycled water, rather than recycling itself.

In the second example, the cost difference to water supply agencies between the Alternative 2b policy and the optimal Alternative 1 policy is presented in Figure 30. These curves are very significant to the water planner in an urban area (Denver, specifically) because they actually represent the value of the dual distributative capacity to the distribution system. For example, consider the condition where effluent BOD and TDS limits are set at 25 mg/l and 800 mg/l, respectively. From Figure 30, a savings of more than 2 million dollars could be realized if the dual systems should be installed in new developments and in the rehabilitation of existing networks. The extent and annual outlay for such construction is not indicated herein and is left to individual planners.

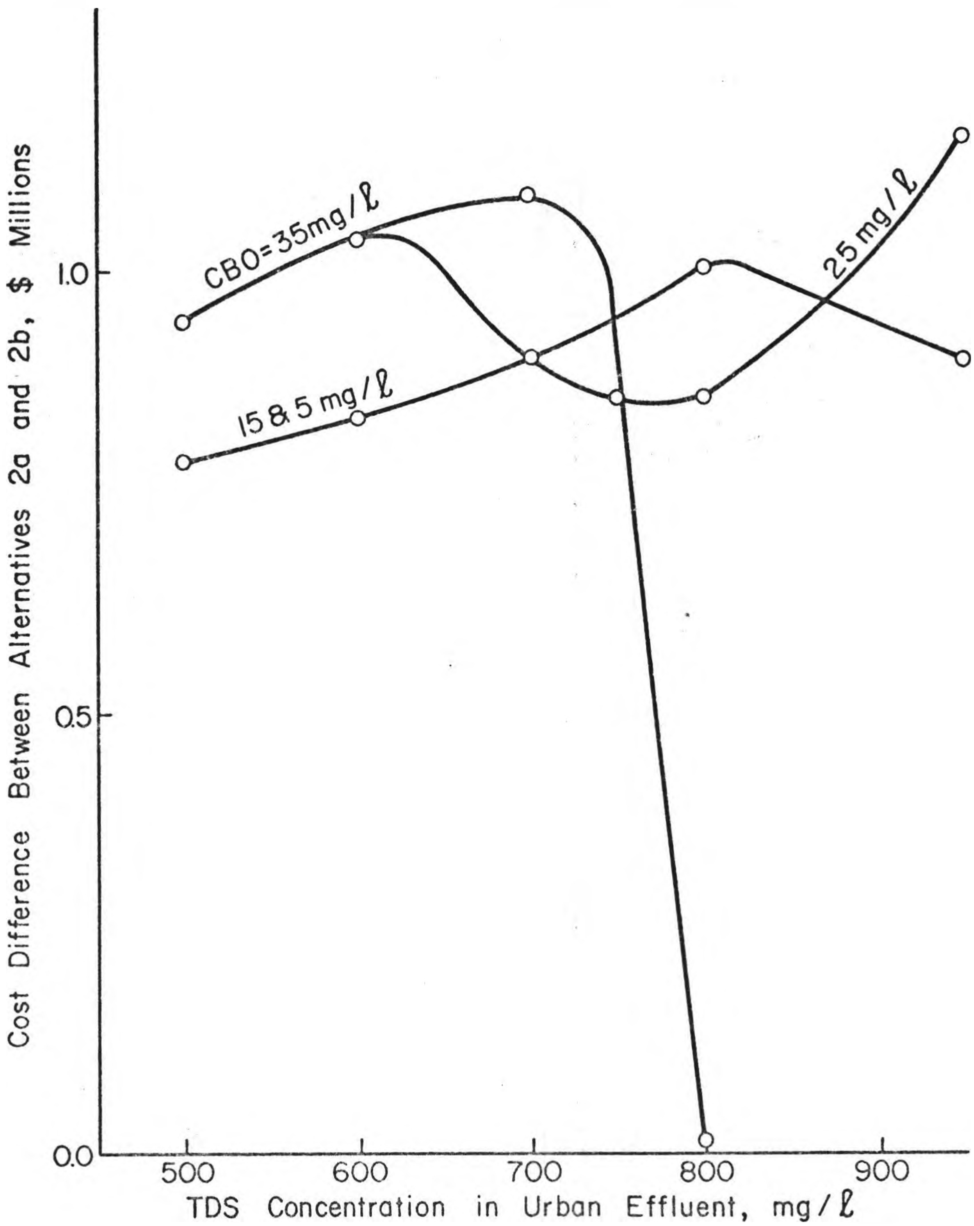


Figure 29. Cost difference between distribution Alternatives 2a and 2b.

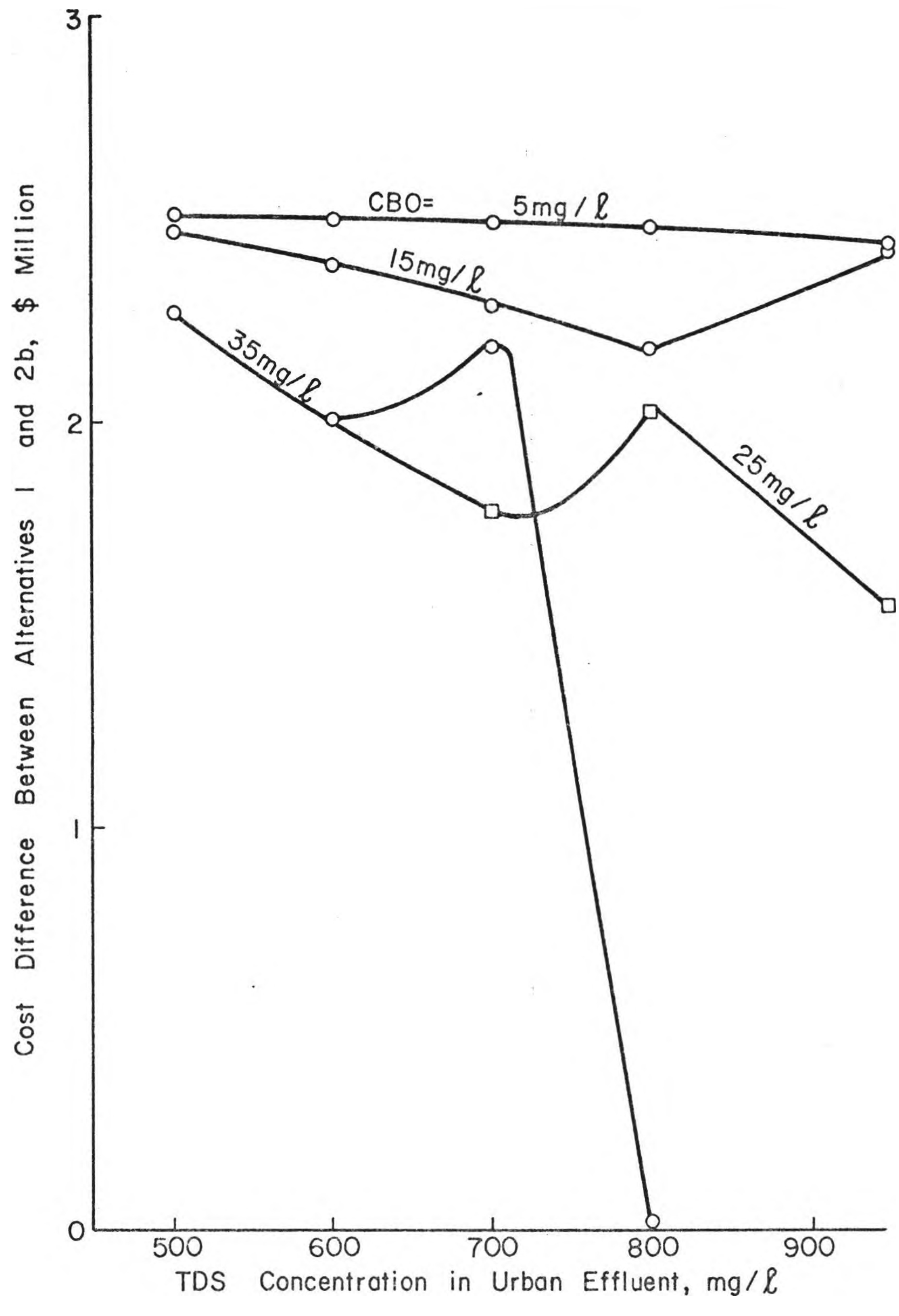


Figure 30. Cost differences between distribution Alternatives 1 and 2b.

Valuing Agricultural Water to Urban Users

One of the most interesting results obtainable from the modeling of urban water management decisions concerns the value that agricultural transfers have in the respective metropolitan uses. In Figures 18, 19, 20, 22, and 23 of the preceding sections, the optimal management policies at different levels of reuse were plotted, and it was clear that significant savings could be realized. However, reuse levels higher than 50% of the imported transbasin diversions would constitute a reuse of other in-basin water rights. Consequently, by varying the level of reuse allowable, it was possible to indirectly evaluate the effect of violating current water rights. Then, by comparing system costs at various levels of reuse, the costs of the right constraint could be determined. In addition, the difference between system costs for the reuse levels also indicate the value of additional inbasin stream flows to the urban user.

In order to demonstrate these results, the value of agricultural water rights to the Denver water user were computed and then plotted in Figure 31. The upper set of curves represent the value of this water to the total urban system when effluent BOD standards are at an anticipated level of about 15 mg/l. There are some apparent contradictions in these curves. First, the value of additional agricultural water decreases as the TDS restrictions on the effluent are relaxed. In fact, if the

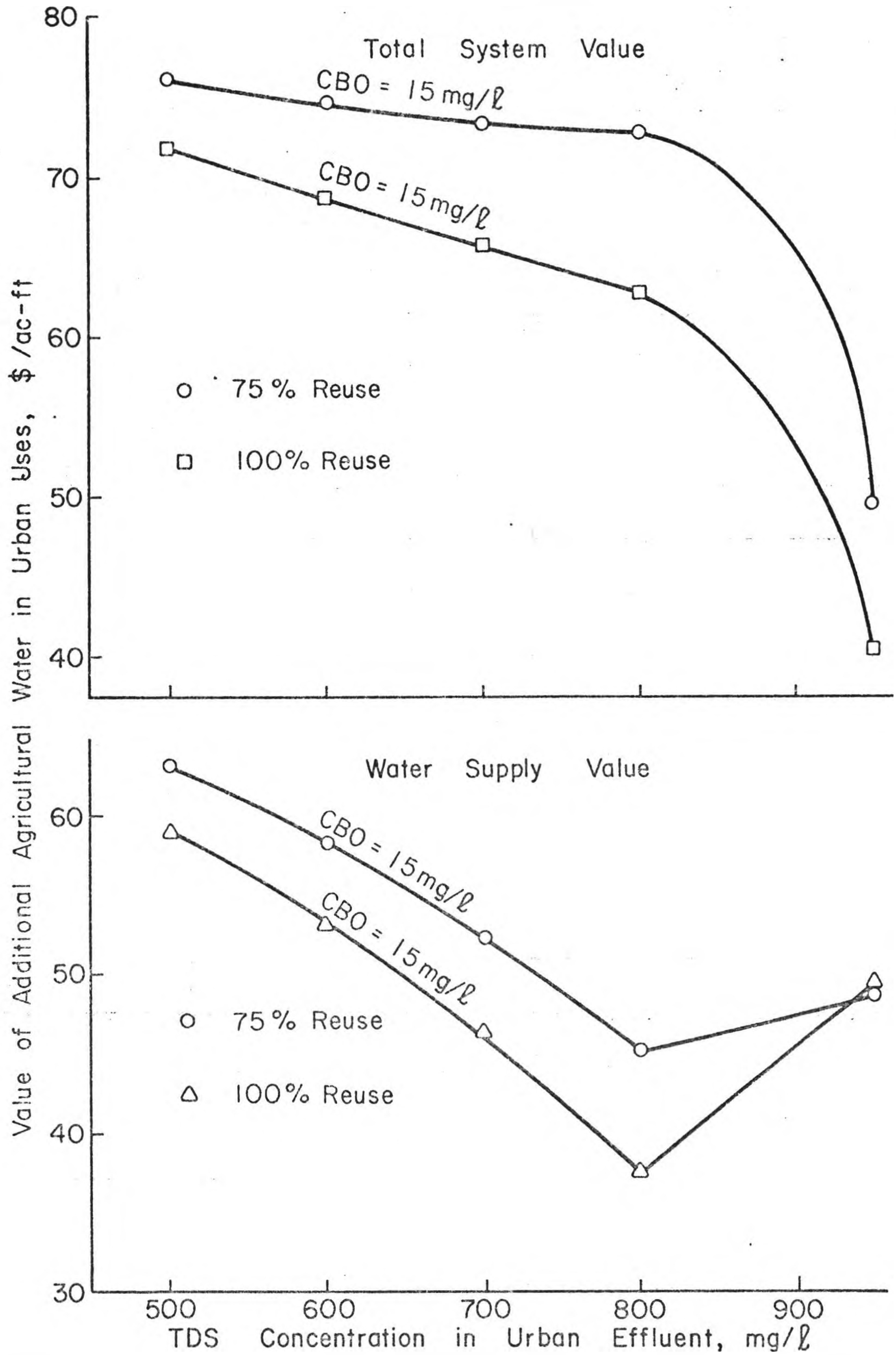


Figure 31. Value of agricultural water right transfers to the metropolitan water user.

constraints are completely relaxed, the value decreases markedly. Keeping in mind that these data were determined from the cost savings derived from allowing higher levels of reuse, the decline in values indicate the decreasing competitiveness of reuse with existing in-stream rights. Thus, as the TDS restrictions are relaxed, the costs of recycling wastewater increase and the overall savings gained from additional reuse are reduced. Another point of interest in these curves is that the 100% reuse level has actually a lower unit value than the 75% reuse alternative, even though an examination of Figures 15 to 20 would point out that the total savings were greater at the 100% reuse level. The curves in Figure 31 illustrate that agricultural transfers have characteristics of diminishing returns.

In the lower curves of Figure 31, the value of the agricultural transfers are shown in terms of the water supply system. The surprising point resulting from these plots is the fact that the supply value is significantly lower than the value to the total urban water system. From Figure 23 and 24 it was noticable that the total savings within the system were greater in the supply portion of the model than in the total system framework. This characteristic is thus another indication of the need to coordinate all aspects of urban water management.

Evaluating the Quality Aspects of Water Rights

Historically, there has been so much concern in the developing river basins of the west for preserving the quantitative aspects of water rights that the qualitative aspects may have been overlooked until the pollution problem reached crisis proportions. For example, the Colorado River Basin has been the scene of quantity conflicts since the early years of the century, but the quality aspects have only recently been under examination. Now, the quality aspects of water utilization may have become the more important consideration.

As new developments occur, or changes in the pattern of use are experienced, effects will undoubtedly be reflected in wastewater salinity levels. In an earlier section, the effects of reuse on effluent TDS levels in the Denver area were shown to vary between 50-250 mg/l, depending on the various parameters characterizing the system. Even with the deficient methods for detecting such changes, it is likely that recycling policies will be constrained by limitations such as those insisting that downstream quality not be deteriorated beyond historical levels. If this is the situation, desalination or mixing with inter-basin transfers must be incorporated within the management strategies. The model was operated in a manner which would yield results illustrating the costs of these institutional constraints. When these costs were evaluated, it was noted that although they did not vary between the various strategies, the results were opposite with respect to water supply costs and total system costs. From the water supply viewpoint, an annual savings actually resulted by maintaining current TDS levels downstream amounting to about 0.5 million dollars annually over the range of effluent BOD concentrations. In the case of the total system, however, the data indicate that restricting TDS in this manner actually cost almost an additional 0.5 million dollars annually.

## Organizational Consolidation

In much of the analysis presented thus far, the emphasis has generally centered around the optimization of both water supply and wastewater disposal operations. It has been tacitly assumed that a linkage between the two sectors could be arranged. However, the present institutional structure demands independently operated systems, which would possibly be inefficient in the future. Governmental management in the Denver area has already attempted to solve problems of this nature when several wastewater treatment systems were consolidated into the Denver Metropolitan Sewage Disposal Distract. A similar consolidation is necessary between the water supply and wastewater handling sectors as well. As with many of the institutional constraints evaluated thus far, such changes may be difficult to achieve but if not successfully attempted, the water users of Denver can expect significant costs for future water resources.

There are three primary areas where consolidation of management and operation responsibilities need to be accomplished in order to implement the strategies in this model. The first is the objective of water supply. Although Denver is by far the largest water supplier (almost 80%), it is necessary to test the feasibility of incorporating all agencies into a single unit. In this manner, the local discrepancies in water supply can be uniformally corrected to give more equitable service throughout the metropolitan area. The model described and used in this work has not included the capability for pricing the effects of this institutional structure although it may be of interest.

The second area for consolidation is in the area of sewage treatment facilities. The Denver area has already accomplished a great deal in this regard with the Regional Service Authority Act of 1972 which allowed the combination of several sewerage agencies into the Denver Metropolitan Sewage Disposal District. The costs associated with not operating in this manner do not only include the facility economics of scale, but also the costs of numerous operational and testing functions. To the extent additional consolidation is possible, it appears to be feasible. As water quality control requirements become more important, centralized treatment is advantageous from both a monitoring, as well as operational viewpoint.

The final consolidation consideration, that of joining water supply and wastewater treatment and disposal function, is of particular importance in the urban water model. Dependence between the two systems is shown as the quantities of water recycled. The costs of the reused water were based on consolidated treatment facilities and would thus be increased by the effects of the scale economics in these structures if the water supply agencies operated their own reclamation plants. The institutional costs of not performing system consolidating thus lies in these scale effects. The cost difference between tertiary and desalting plants in the model and those for separate systems for a typical situation in Denver would be about 0.5 - 1.0 million dollars annually. Thus, multipurpose treatment systems represent savings of 5 - 10% of the total annual water supply costs.

## Projecting Present Analyses to Future Policies

Since the events expected to occur in the future cannot be assessed in a reliable fashion, the approach taken in this writing is to exhaustively examine current management decisions and exogeneous constraints on the behavior of optimal policies. Then, by means of extending this investigation towards future conditions, the foundations for rational decisions can be laid as they become necessary.

## Optimal Policy Spaces

An analysis of decision points identified in the modeling revealed that the future policy spaces are almost identical with current ones. Thus, Figures 15, 16, and 17 are fair representations of optimal management schemes in the years to come. This may be initially surprising until it is recalled that the only changes made in the model inputs were demand increases. Since these were also proportionately the same, the optimal policy spaces could be expected to be the same. With homogeneous policy spaces, the shape of cost curves could also be expected to remain similar. An examination of the results in this regard substantiated this hypothesis.

## Future Costs

As a means of expanding current conditions and costs to a future date, the ratio of present costs and the results of a future analysis was determined. Beginning first with the water supply costs as functions of time, the ratios for various times were computed for Alternative 2b with a fixed TDS level of 800 mg/l. The results of these calculations are shown in Figure 32. The variance reflected for different levels of water quality in previous plots is almost absent here. One interesting point can be made, however. Notice that the supply ratio decreases slightly with BOD concentrations. This characteristic follows the analysis of total water supply

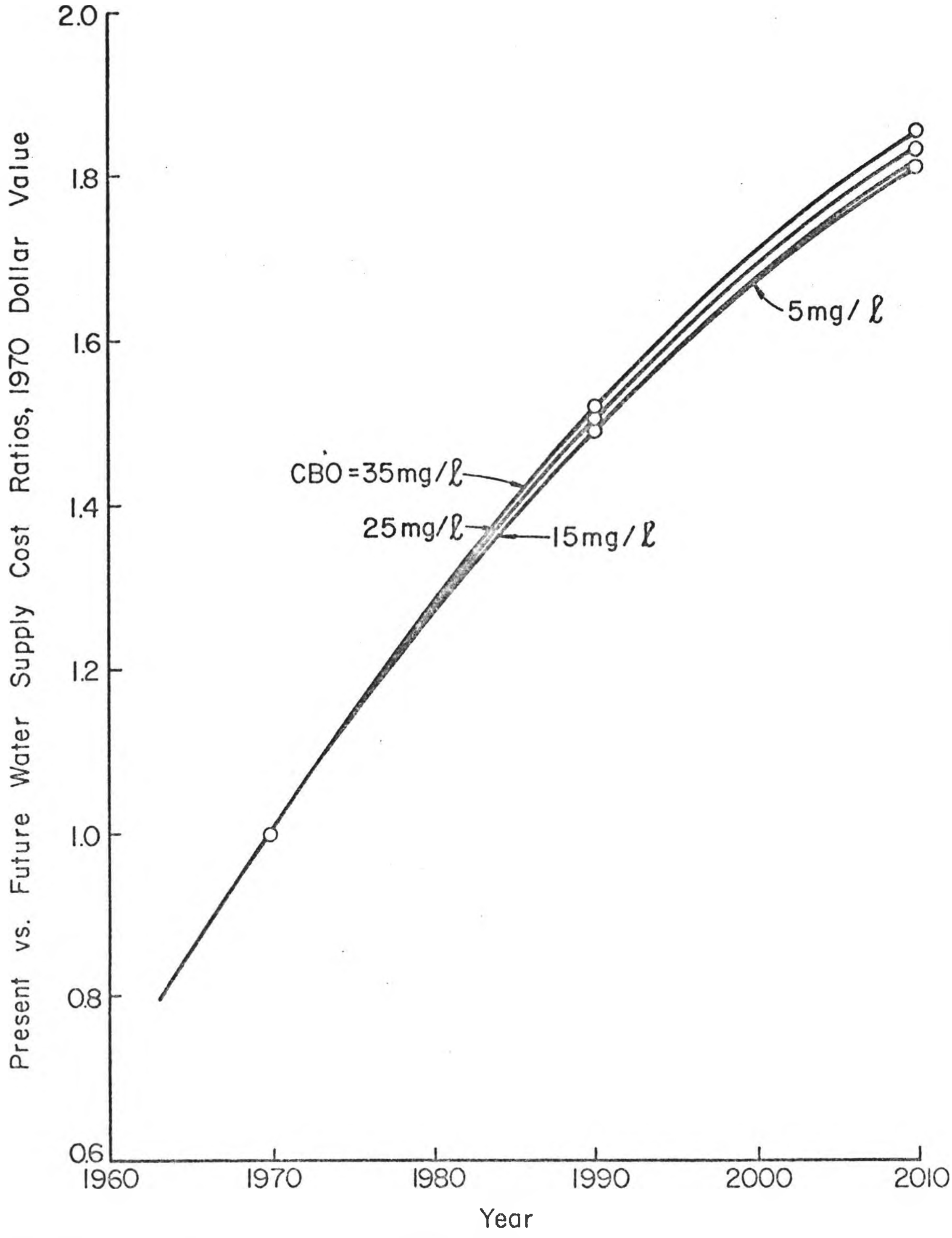


Figure 32. Ratios of present water supply costs to those expected in the future for Alternative 2b at a fixed effluent TDS level of 800 mg/l.

costs, which substantially decreased when more restrictive water quality controls were imposed upon the effluent.

This analysis was repeated for wastewater reclamation and treatment costs for Alternative 2b, which is presented in Figure 33. The condition is again similar to the change in total costs of wastewater treatment as a function of water quality produced. Almost no variation was found when ratios were computed for total system costs. As a result, the curves presented here can be used as an index to expected costs in the future. All data analyzed were in terms of 1970 dollar values and do not account for inflationary trends. These curves demonstrate the economies of scale associated with constructing the facilities to accomplish optimal water management strategies.

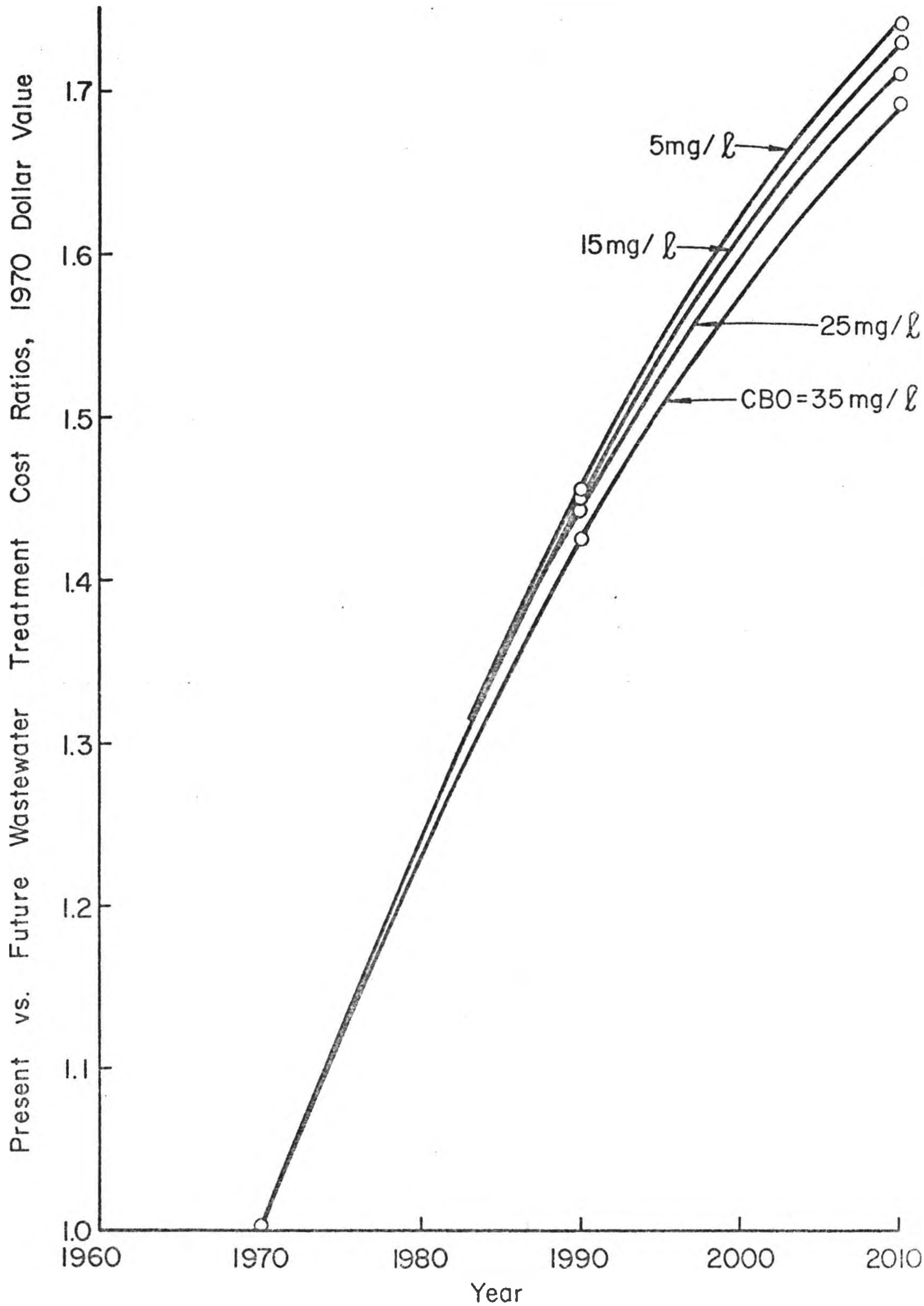


Figure 33. Present verses future wastewater treatment cost ratios for Alternative 2b at a fixed effluent TDS standard of 800 mg/l.

## Chapter 7

# SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

### Introduction

The objective of this work has been to provide a tool which would be useful in evaluating the degree with which institutional factors can be expected to hinder changes towards optimal water management strategies. This study has been specifically oriented towards arid urban centers since they best characterize the conditions of water scarcity. Most notably, the example of the Denver Colorado area was selected to demonstrate the analysis proposed herein. This chapter is intended to summarize the major conclusions reached and the suggested research and evaluation which could be initiated to extend these results.

### Summary

The complicated urban water system comprising physical facilities, legal, social, and political rules for operation can be divided broadly into three main subsystems: (1) water supply; (2) water demands; and (3) wastewater treatment. In order to accomplish the objective of this study, the scope of the investigation was limited to an examination of the interrelationships existing between these subsystems. A mathematical model describing the basic operation of the urban water system was developed from which an array of management strategies were tested. Then, an imputation analysis on these results was made to evaluate the system characteristics, costs of institutional constraints, and future strategies. Some of the findings of this study are summarized below.

Scope of Modeling

The scope of any model should be governed by the questions it is designed to answer. The magnitude and complexity of urban water systems are such that a single model capable of providing answers to each detail of its operation would be an unrealistic objective. Separate institutional structures which manage individual segments of the systems and the physical complexity of the system usually limit model development to a description of a particular aspect in order that answers to selected questions can be generated. The apparent weakness in this philosophy is that an assumption must be made regarding the constancy of external variables which may in reality be affected by changes in the limited scope model. Such a weakness, however, can be readily remedied by operating two or more such models in a multilevel optimization format or by simply performing an exhaustive sensitivity analysis.

The goals of this study centered around optimizing urban water management strategies to test the constraining nature of various institutional factors on future water management decisions. Consequently, the detail of the model was judged to be sufficient at the level of the three primary subsystems noted above. For example, Denver has numerous water supply systems drawing water from stream flows and interbasin transfers, but the scope of this model considers the total water supply as being delivered by a single organization. This hypothetical water supply agency thus has water resource alternatives the same as the existing agencies, but only a one-dimensional decision process. In a similar manner, the water demands were consolidated into categories of domestic, municipal, and industrial with the wastewater treatment subsystem being subdivided into unit operations.

The integration of the management decisions in each of the three primary subsystems into a single model provided both a forward loop (supply-distribution-wastewater) and a backward loop (wastewater-supply-distribution) interrelationship. Both of these coordinations, achieved by adding reuse to the water supply alternatives, permits the evaluation of the effects in one subsystem resulting from changes in another subsystem.

This modeling scope permits the initial delineation of system characteristics which are common to each management policy, and then imputing changes in these parameters by comparing modeling results. For instance, the costs of agricultural water right transfers in the water supply sector, as well as the distribution system costs in the demand sector, were temporarily omitted from the modeling results but later evaluated. By operating the model in this manner, the results are indicators of the relative feasibility of the alternatives. However, when the differences between various system alternatives are obtained, the common element factors drop out and the results are real values of the isolated parameter.

The scope of this work allows a great deal of flexibility in the analytical phase to explore important questions regarding current and future urban water management strategies.

## Model Optimization

Urban areas which face water shortages and tighter wastewater quality standards must choose between several alternatives to accomplish the requirements imposed. The best choice is seldom obvious in light of events which could occur in either the immediate or long range futures. Consequently, a systems analysis method is employed to evaluate the optimum strategy to be followed.

Optimization usually follows from a mathematical description of the alternatives. In this model, a minimum cost criterion has been selected upon which alternatives can be ranked. No pretense is made to assume minimum cost achieves regional, state, or national optimization of water resource utilizations, but it is concluded that this analysis yields the best indication of water management within the urban setting. The combination of economic indicators for each segment of the urban water system, uniquely determined for each alternative, formulates the objective function which is the essential element of any optimization process. Certain portions of the objective function, however, must be restricted to some range of values to reflect constraints on the alternatives which exist in the study. Thus, optimization is accomplished subject to these constraining relationships.

The urban water system is in reality a non-linear system. The costs exhibit both increasing and decreasing marginal costs as the scale of facilities increase. In addition, the interrelationship between parameters in the system (e.g., water quantity and quality) makes many of the constraining functions non-linear as well. Several optimum seeking techniques are available for use in a problem of this nature, but most methods require considerable modification and great care in their use. Furthermore, optimization techniques which are modified for use in special problems often lack either generality or flexibility. In this study, a general non-linear differential algorithm was programmed. This method is based on the basic theorems of differential calculas. The theoretical aspects of the algorithm are relatively straightforward, but the operational computer code is highly complex, thereby limiting its utility as compared with the simpler methods.

## Conclusions

The results of this analysis do not identify any specific inefficiencies in Denver's present water management policies, but they do suggest radical departure in the immediate future. Much of the current planning deals with expanding the capacities of interbasin transfer systems to satisfy a growing need, while increasing the capability of wastewater treatment operations to accomplish more refined pollutant removal as dictated by federal policies. The principal conclusion of this work is that these two aspects should be coordinated under the objective of urban water management improvement. In the following paragraphs some of the specific conclusions are presented which illustrate the necessity for this coordination.

In this model, the costs of both in-basin stream flow diversions and interbasin transfers have been linearized at values suggested by various planning agencies. However, recycling costs are non-linear functions characteristic of wastewater treatment and reclamation costs. This mix yields some interesting results. First, the policy space indicating the optimal decisions as functions of important model parameters is sharply divided into three primary regions of reuse: (1) zero reuse; (2) limited reuse; and (3) unlimited reuse. These regions indicate the relative feasibility of recycling urban effluent as water quality constraints are changed. Secondly, the cost functions expressing both water supply costs and total system costs change shape abruptly when the model parameters are varied in such a manner as to identify policies in two or three of the distinct regions of the policy spaces. Finally, the fixed cost nature introduced by the linearities make the optimal decisions dependent almost exclusively on the quality standards set for the urban effluents. Although this was an unintentional result, it was nevertheless helpful in this study since the effects of water pollution controls on urban water management decisions were among the answers sought. As functions of these parameters, water supply costs under optimal policies were shown to substantially decrease as effluent standards were made more stringent. However, the total system costs were increased as these restrictions were applied. The added costs of wastewater treatment were therefore sufficient to offset the savings in water supply. Optimal policies provided substantially lower costs than the existing mode of operations in all of the model analyses.

Three alternative distribution systems were proposed and compared. The first involved forcing reuse to take place through existing raw water facilities. Optimal water source selections were based on the relative costs of water from the potential water supply sources. The remaining two alternatives involved recycling directly to individual demands in the system, thereby comparing water sources as treated supplies. Because the first alternative increased the water costs by the raw water treatment expenditures, the last two schemes showed significant savings. The first of the two final alternatives maintained TDS concentrations at a domestic standard (300 mg/l in this study) while the second allowed increased TDS concentrations in both municipal and industrial uses. If water becomes a scarce and valuable resource, a priority must be placed on not only quantity but quality as well. Various uses certainly are more important than others (e.g., drinking water versus lawn watering) and similarily some uses are more sensitive to water quality (same example). Consequently, the last alternative for water distribution is concluded to be the best for the future. Such a policy requires dual distribution systems in the urban area to satisfy the various demands. The costs of these changes were not evaluated, but the water supply savings were imputed and suggest the feasibility of transforming the system by adding dual systems as part of rehabilitation projects and new developments. The costs of public attitudes were evaluated by comparing the three alternative distribution strategies and were shown to be most significant. Certainly, if the funding is not declared by the voters, nothing in the way of a structural change can be accomplished.

The use of recycled water in the urban system will generally be accomplished at water quality concentrations substantially above existing supplies in order for the costs of these flows to be competitive with other sources.

By mixing recycled water with stream flow and importations, the basic water quality criteria for each type of demand can still be met, but at lower costs. However, if the inputs to the metropolitan water use reflect increases in TDS concentrations, for example, it is reasonable to conclude that such changes will be manifested in the wastewater flows. A simple proportionality relationship was used to identify these changes in the model but was weighted to account for the difference between the consumption ratios of the individual demands. Then, the costs of maintaining existing effluent levels of TDS were compared to a policy of simply allowing them to increase. These results did not show marked changes in the form of the optimal policies, but were useful in reevaluating the costs of the qualitative aspects of water rights.

Water rights developed a century or more ago to equitably allocate and protect water resources have seen few modifications which would reflect the evolution of water resource needs. Early water resource developments were undertaken primarily to supply agricultural requirements. Subsequent expansions in urban demands should have been met at least partially with agricultural transfers, which could have been achieved by improved irrigation practices. Such transfers have only been successful to a small degree because of the water right laws. If revisions are made in these statutes in the future, then it may be feasible for metropolitan planners to initiate procedures for obtaining agricultural water transfers. In order to evaluate the feasibility of such transfers, the levels of reuse were extended beyond the interbasin transfer residuals to include a substantial fraction of the return flows from other sources. Since these additional return flows are used to satisfy downstream water rights, a comparison between the various levels of reuse indicate the value of these quantities of water to the Denver user. The results illustrate urban water values two or three times greater than the value to agriculture. However, it is debatable whether or not such figures suggest an economic necessity to transfer water from agricultural to urban uses, and it must remain so even in view of these results since a host of socio-political considerations have been omitted.

The future growth of Denver was evaluated in the model but may be less influential than technological advancements, unexpected population shifts, or other unforeseen events. The results are therefore indications of optimal policies projected from present conditions and are useful in evaluating system behavior resulting from inputs which seem likely to occur. In spite of the limitation of not adequately assessing future events, the need to coordinate water supply, distribution, and wastewater treatment to satisfy future water demands seems clear.

If one conclusion was to be singled out as particularly significant, it would be that the effect of increasingly stringent water quality standards is to promote the feasibility of recycling reclaimed wastewater.

## Recommendations

### Model Limitations

There were three simplifications made in this study which were permissible without substantially weakening the model. However, improved information would allow the investigation of other important questions. The first of these assumptions dealt with the prediction of TDS increases which would occur as a result of supplying the urban demands with a poorer quality water. From a simple mass balance, a significant "pick up" effect is noticable in the Denver area. As reuse becomes widely accepted, this question needs to be resolved since water short regions generally have salinity problems. To avoid further inequitable downstream damages resulting from using a more degraded water supply, steps should be taken to predict the magnitude of the problem and suggest remedies. In terms of this model, it would be helpful to delineate the TDS effects with respect to the type of urban demand so that improved predictions could be facilitated.

The second simplification was the assumption that the costs of interbasin transfers are linear, when in fact they probably exhibit some economy of scale. In the Denver situation, these importations come from the Upper Colorado River Basin, a region where salinity is at critical levels.

Thus, the removal of high quality water from the watershed reduces the dilution capacity of downstream flows. To avoid additional downstream damages, it is likely that each increment of interbasin transfer must be accompanied by a salinity control measure in the Colorado River Basin. These salinity control costs are definitely nonlinear and should be reflected in the costs of the imported water. A study should be undertaken to evaluate an optimal level of interbasin transfers in this region. This would introduce the interregional dimension to water management planning.

The final aspect of the model that should be modified is the addition of other water quality parameters to the analysis. Both phosphorous and nitrogen are important parameters in terms of the algae bloom problems. In addition, some heavy metals introduced by an urban system, or some other exotic pollutant, may be of importance in a specific study.

## New Technology

Each aspect of the urban water system comprises a major field of engineering interest. Wastewater treatment, for example, is extensively investigated and applied by engineers specializing in sanitary and environmental engineering. Others, such as this author, draw from these disciplines in generating models. It is necessary when attempting to perform such research to rely on the more informed judgment of the specialists to present information in a manner usable by others outside the field.

Among the essential needs in effective modeling of water management strategies in urban areas concerns wastewater treatment. New and more refined processes are being developed, tested, and applied continually to the pressing problems of water pollution. The results of this study show these advancements and applications need to be coordinated with water supply and distribution to demands if water management is to be effective. Consequently, there is a need for investigators to convert the highly technical information concerning wastewater treatment processes to information usable by planners not entirely conversant in the details of treatment processes. The information used in this study regarding the costs of constructing treatment facilities is an excellent example of such work. What is lacking, however, is alternative technologies and periodic improvements that could be included in such work. It is therefore recommended that the engineering profession continue to increase its efforts to make their knowledge usable in broader scope research by other disciplines.

In employing the best available information in evaluating alternatives, systems analysis concepts are used to coordinate a search for the optimal solution, given the circumstances of the study. The mathematical methods for optimization form the basis for the discipline of operations research, which is of relatively recent origin, especially as applied to the field of water resources. As a result, applications have only recently emerged from a novice state to one of practical use. This is demonstrated by the shift from large all-inclusive models to smaller limited-purpose models coordinated by special methods such as multilevel optimization. The disadvantage of the large models is the absolute lack of generality. The smaller limited-purpose models are more usable by other individuals in applied practice than are the more sophisticated models, but require more experienced judgment. Thus, more emphasis should be placed on constructing models for solving real management problems.

A final recommendation for improving the implementation of new technology in problem evaluation and solution concerns the multidisciplinary aspects of current studies. Political, social, legal, economic, and engineering studies are being undertaken to provide direction for subsequent decisions, but very few of these are using the most recent findings from the disciplines. Just as wastewater treatment technology needs to be transcribed into a usable form for all individuals lacking the necessary expertise, the results of various studies by other disciplines need disemination to wider readership. However, it is probably more realistic to incorporate these disciplines into specific study teams because of the vastly different trainings acquired by professionals in these fields. Consequently,

water resource planning should not only be multidiscip-
linary, but interdisciplinary as well.

## REFERENCES


1. Albertson, M. L., Taylor, D. C., and Tucker, L. S. 1971. Summary--The Urban Water System. Section XII, Treatise of Urban Water Systems, Ed. by M. L. Albertson, D. C. Taylor, and L. S. Tucker. Colorado State University, Fort Collins, Colorado. July.

2. Alexander Patter Associates. 1970. Review and Evaluation of HDR Report on Metro Plant Expansion Study. Metropolitan Denver Sewage Disposal District No. 1. Denver, Colorado. January.

3. Anderson, N. W., and Day, H. J. 1968. Regional Management of Water Quality: A Systems Approach. Journal of the Water Pollution Control Federation, Vol. 40, No. 12, October.

4. Banks, H. O., Inerfield, A. J., Wolfe, C. G., Tucker, D. L., and Harris, P. G. 1971. Economic and Institutional Analysis of Wastewater Reclamation and Reuse Projects. Leeds, Hill, and Jewett, Inc., Consulting Engineers, San Francisco, California. December.

5. Barnard, J. L., and Eckenfelder, W. W. 1970. Treatment-Cost Relationships for Organic Industrial Wastes. Paper presented at the Fifth International Water Pollution Research Conference. August.

6. Bishop, A. B., and Hendricks, D. W. 1971. Water Reuse Systems Analysis. Proceedings of ASCE, Journal of the Sanitary Engineering Division, Vol. 97, No. SA1, Paper 7898. February.

7. Board of Water Commissioners. 1971. Annual Statistical Report. Denver, Colorado. December.

8. Board of Water Commissioners. 1972. 1972 Planning Program. Denver, Colorado. May.

9. Case, F. E. 1972. Economics of Water Quality and Wastewater Control. Proceedings of ASCE, Journal of the Sanitary Engineering Division, Vol. 98, No. SA2, Paper 8856. April.

10. Civil Engineering. 1972. Reuse of Treated Wastewater as Potable Water. Summary of papers presented at the 1972 Environmental Meeting, ASCE. p. 46. December.

11. Coe, J. J. and Laverty, F. B. 1972. Wastewater Reclamation in Southern California. Proceedings of ASCE, Journal of the Irrigation and Drainage Division, Vol. 98, No. IR3, Paper 9178. September.

12. Coffey, P. J., and Ortolano, L. 1969. Economic Framework for Salinity Control Projects. Journal of the American Water Works Association, Vol. 61, No. 5, May.

13. Crawford, I. C. 1957. Water Resource Planning in Colorado. Colorado Water Conservation Board. Denver, Colorado. September.

14. Denver Regional Council of Governments. 1972. An Approach to Regional Service, The Colorado Service Authority Act of '72. Report COLO.P-96. Denver, Colorado. May.

15. Denver Water Department - Consultant. 1969. Metropolitan Water Requirements and Resources 1968-2010. Vol. 1 Text and Vol. 2 Technical Appendices. Submitted to Denver Regional Council of Governments. Denver, Colorado. July.

16. Dérédec, A. 1972. A Systematic Approach to the Water Supply of a Large Urban Area. M.S. Thesis, Civil Engineering Department, Colorado State University, Fort Collins, Colorado. May.

17. Dracup, J. A., Mobasheri, F., and Cardenos, M. A. 1970. An Assessment of Optimization Techniques as Applied to Water Resource Systems. Environmental Dynamics, Inc., Los Angeles, California. August.

18. Dykstra, D. I. 1968. Status of Office of Saline Water Development in Brackish Water Conversion. "Water 1968," Vol. 64, No. 90, American Institute of Chemical Engineers.

19. Eckenfelder, W. W., and Adams, C. E. 1972. Design and Economics of Joint Wastewater Treatment. Proceedings of ASCE, Journal of the Sanitary Engineering Division, Vol. 98, No. SA1, Paper 8746. February.

20. Evans, D. R. and Wilson, J. C. 1972. Capital and Operating Costs - Advances Wastewater Treatment. Journal of the Water Pollution Control Federation, Vol. 44, No. 1. January.

21. Fair, G. M., Geyer, J. C., and Okun, D. A. 1968. Water and Wastewater Engineering, Vol. 2, Water Purification and Wastewater Treatment and Disposal. John Wiley & Sons, Inc. New York, New York.

22. Federal Water Pollution Control Administration. 1966. Report to the Second Session of the Conferences in the

Matter of Pollution in the South Platte River Basin. United States Department of Health, Education, and Welfare. South Platte River Basin Project. Denver Colorado. April.

23. Flack, J. E. 1971. Allocation of Resources with a Case Study of the Water Management Game. Section V, Treatise on Urban Water Systems, Ed. by M. L. Albertson, D. C. Taylor, and L. S. Tucker. Colorado State University, Fort Collins, Colorado. July.

24. Frankel, R. J. 1965. Water Quality Management: Engineering-Economic Factors in Municipal Waste Disposal. American Geophysical Union, Water Resources Research, Vol. 1, No. 2. Second Quarter.

25. Hall, W. A. and Dracup, J. A. 1971. Water Resources Systems Engineering. McGraw-Hill, Inc., New York, N.Y.

26. Hall, W. A., Haimes, Y. Y., and Butcher, W. S. 1972. Analysis of the Feasibility of Interim Water Supplies. American Geophysical Union, Water Resources Research, Vol. 8, No. 2. April.

27. Hartmen, L. M. and Seastone, D. 1970. Water Transfers, Economic Efficiency and Alternative Institutions. The John Hopkins Press, Baltimore, Maryland.

28. Hass, J. E. 1970. Optimal Taxing for the Abatement of Water Pollution. American Geophysical Union, Water Resources Research, Vol. 6, No. 2. April.

29. Henningson, Durham, and Richardson, Consultants. 1970. Area Wide Sewerage Master Plan Report for the Denver Metropolitan Area, Phase I. Submitted to the Denver Regional Council of Governments, Denver, Colorado. March.

30. Hobbs, N. 1971. Case Study on Linear Programming for Optimization of Production and Supply. Section VII, Treatise on Urban Water Systems, Ed. by M. L. Albertson, D. C. Taylor, and L. S. Tucker. Colorado State University, Fort Collins, Colorado. July.

31. Horowitz, A. J. 1970. Optimization of Water Quality Systems by Non-linear Programming. M.S. Thesis, School of Engineering and Applies Science, University of California, Los Angeles, California.

32. Illinois State Water Survey. Cost of Water Treatment in Illinois. October. ( Cited by Dérédec, 1972)

33. Iorns, W. V., Hembree, C. H., and Oakland, G. L. 1965. Water Resources of the Upper Colorado River Basin - Technical Report. Geological Survey Professional Paper 441. U. S. Government Printing Office, Washington, D. C.

34. Kneese, A. V. 1964. The Economics of Regional Water Quality Management The Johns Hopkins Press, Baltimore, Maryland.

35. Kneese, A. V. and Bower, B. T. 1968. Managing Water Quality: Economics, Technology, Institutions. The Johns Hopkins Press, Baltimore, Maryland.

36. Kuhn, H. W. and Tucker, A. W. 1951. Nonlinear Programming. Proceedings of the Second Berkeley Symposium on Mathematics, Statistics and Probability. J. Neyman, Ed. University of California Press, Berkeley, California.

37. Mencken, H. L. 1942. A New Dictionary of Quotations. Alfred A. Knopf. New York, New York.

38. Michel, R. L. 1970. Costs and Manpower for Municipal Wastewater Treatment Plant Operation and Maintenance, 1965-1968. Journal of the Water Pollution Control Federation, Vol. 42, No. 11. November.

39. Monarchi, D. 1972. Interactive Algorithm for Multiple Objective Decision Making. Technical Report No. 6, Technical Reports on Hydrology and Water Resources. University of Arizona, Tucson, Arizona. June.

40. McGauhey, P. H. 1966. Quality Aspects of Ground Water Use and Management. Proceedings of the 1965 Biennial Conference on Ground Water Recharge, Development, and Management. Fresno Field Station, Agricultural Research Service, U.S. Department of Agriculture. January.

41. National Water Commission. 1972. Interbasin Water Transfers - A political and Institutional Analysis. National Technical Information Service, U.S. Department of Commerce. Springfield, Virginia. March.

42. Nichols, S. R., Skogerboe, G. V., and Ward, R. C. 1972. Water Quality Management Decisions in Colorado. Completion Report Series, No. 38. Environmental Resources Center, Colorado State University, Fort Collins, Colorado. June.

43. Orlob, G. T. 1971. The Urban Water System - Techni cal Aspects. Chapter ID, <u>Treatise on Urban Water Systems</u>, Ed. by M. L. Albertson, D. C. Taylor, and L. S. Tucker. Colorado State University, Fort Collins, Colorado. July.

44. Pavone, A. ]971. Comprehensive Public Water Supply Planning. Proceedings of ASCE, Journal of the Sanitary Engineering Division, Vol. 97, No. SA5, Paper 8450. October.

45. Rambow, C. A. Industrial Waste Water Reclamation. Paper presented at the 23rd Purdue Industrial Waste Conference, Lafayette, Indiana. (Cited by Dérédec,1972)

46. Reid, G. W. 1971. The Macro Approach - Urban Water Demand Models. Section IV, <u>Treatise on Urban Water Systems</u>, Ed. by M. L. Albertson, D. C. Taylor, and L. S. Tucker. Colorado State University, Fort Collins, Colorado. July.

47. Schierbrock, W. N. 1960. The Denver Site, A Geographic Study. M.S. Thesis, Graduate College, University of Denver, Denver, Colorado.

48. Shah, K. L. and Reid, G. W. 1970. Techniques for Estimating Construction Costs of Waste-Treatment Plants. Journal of the Water Pollution Control Federation, Vol. 42, No. 5. May.

49. Shiozawa, S. 1972. Dual-purpose Desalting in Pacific Southwest. Proceedings of ASCE, Journal of the Hydraulics Division, Vol. 98, No. HY1, Paper 8673. January.

50. Smith, E. T. and Morris, A. R. 1969. Systems Analysis for Optimal Water Quality Management. Journal of the Water Pollution Control Federation, Vol. 41, No. 9. September.

51. Smith, R. 1968. Cost of Conventional and Advance Treatment of Wastewater. Journal of the Water Pollution Control Federation, Vol. 40, No. 12. September.

52. Thomann, R. V. 1972. <u>Systems Analysis and Water Quality Management</u>. Environmental Science Services Division, Environmental Research and Applications, Inc. New York, New York.

53. Todd, D. K. 1960. <u>Ground Water Hydrology</u>. John Wiley & Sons, Inc. New York, New York.

54. U.S. Environmental Protection Agency. 1972. Conference in the Matter of Pollution of the Interstate Waters of the Colorado River and its Tributaries - Colorado, New Mexico, Arizona, California, Nevada, Wyoming, Utah. Proceedings of the Reconvened Seventh Session. Denver, Colorado. April.

55. U.S. Environmental Protection Agency. 1972. Effects of Waste Discharges on the Water Quality of the South Platte River, Denver Metropolitan Area. National Field Investigations Center - Denver and Region VIII. Denver, Colorado. June.

56. White, K. R. 1971. Improving Municipal Water Supplies by Desalting. Prepared for the Colorado Water Conservation Board and Office of Saline Water, U.S. Department of the Interior. Denver, Colorado. April.

57. Wilde, D. J. and Beightler, C. S. 1967. Foundations of Optimization. Prentice - Hall, Inc., Englewood Cliffs, New Jersey.

# APPENDIX A

## OPERATION OF THE URBAN WATER SYSTEM MODEL

### Introduction

In order to clarify the transistion between the rigorous mathematical derivations of Chapters 3, 4, and 5, and the mechanical facets of the model's operation, this short appendix is presented.

The urban water system model developed in Chapters 4 and 5 consists of two coordinated subsystems. These are the urban water supply and distribution, and the urban wastewater treatment and reclamation systems. Functions describing the costs of these subsystems and constraints controlling their operation are incorporated in the Jacobian Differential Algorithm where optimization of water management strategies is facilitated. The general urban water system model defined in the introductory chapter is thus formulated as a minimization problem in the algorithm with a small exterior program to supply data and an initial feasible solution.

### Model Operation

The model is divided into two segments which represent the water supply and wastewater treatment phases of the urban system with the distribution phase formulated in the constraining functions. The execution of the model is a two step iterative procedure in which the facility costs of the two model segments are minimized in accordance with the distribution system.

The first step in optimizing the system is to compute the relationship between reuse costs and the concentration of TDS in these flows for the conditions imposed on the urban effluents. This is accomplished by determining a series of unit differences between an optimized wastewater treatment system for two levels of reuse ( zero and a specified value). Then a polynomial regression is performed in which the reuse cost relationship is evaluated.

With the reuse cost function, the second step compares the economic feasibility of the array of water supply alternatives and selects the one having the least cost while meeting the physical realities of the system. Then the levels of reuse are contrasted with the estimates employed in the first step to evaluate recycling costs. If a descrepancy exists, the revised estimate is submitted to the optimization in step 1 and a new iteration is completed. Eventually, the descrepancies are eliminated as the procedure converges and the entire urban water system is optimized.

In any specific problem, the only parameters allowed to vary are the flows obtained from the respective water sources, the mixing flows in the wastewater treatment model, and the quantities and qualities of the reuse. As a result, the inputs to the model include:

(1) magnitude of individual urban demands

(2) unit costs of the stream flows and inter-basin transfers

(3) water quality standards imposed on the urban effluents

(4) water quality standards imposed to protect individual urban water demands

(5) an initial feasible solution for both suboptimizations

These requirements are met for a wide range of conditions by the exterior input program.

## Model Formulation

Since the model is a minimization problem of the differential algorithm, the cost functions for each segment of the urban water system are formulated as objective functions in Subroutine YOFX. The constraints which insure that the flows are meeting the requirements of the demands and effluents are defined in Subroutine FKOFX. Derivatives of each cost function or constraint with respect to the model variables are defined in Subroutines DYDX and DFDX respectively. These subroutines were defined in Table 1.

The cost functions are taken from Equations 43, 46, 49, 53, 54, 55, 56, 57, 58, 59, and 60 of Chapter 4 and Equations 75, 79, and 81 of Chapter 5 for the two model segments. These functions are converted to $ million in terms of 1970 dollars and then amortized over a 30 to

50 year repayment period at a discount rate of 5-1/2%. The objective functions for the two model segments can thus be written,

$$y_1 = \min \sum_{j=1}^{4} P_j \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (83)$$

and,

$$y_2 = \min \sum_{j=1}^{4} R_j \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (84)$$

where $y_1$ and $y_2$ are the annual costs of the water supplies (including raw water treatment) and the wastewater treatment costs (at the two levels of recycling) respectively in $ million annually, and $P_j$ and $R_j$ are the amortized cost functions for the facilities summarized in Tables 2 and 3.

The constraints on the problem are the same expressions defined in Chapters 4 and 5 and these will not be reiterated here.

Table 2. Cost functions for the wastewater treatment system.

$$P_1 = 11.7CZ_1^{.74}$$

$$P_2 = 18.4CZ_1^{.76}$$

$$P_3 = 4.2CZ_2^{.95} + 14.8CZ_2^{.69} + \begin{cases} 3.3CZ_2^{.57} & \text{if } Z_2<3\text{mgd} \\ 1.8CZ_2^{.94} & \text{if } Z_2>3\text{mgd} \end{cases}$$

$$P_4 = 0.3\left[\frac{1.0-T}{T}\right]^{.58} 20.0CZ_3^{.82}$$

C = time adjustment factor ( to 1970 dollars)
T = desalting TDS removal efficiency
$Z_1$= capacity of primary and secondary wastewater treatment plants, mgd
$Z_2$= capacity of tertiary treatment plant, mgd
$Z_3$= capacity of desalting plant, mgd

Table 3. Cost functions for the water supply system.

$$R_1 = 100.0X_1$$

$$R_2 = 10.0X_2$$

$$R_3 = 200.0CZ_4^{.74}$$

$$R_4 = (X_7 + X_9)(A_1 + A_2C_{tr} + A_3C_{tr}^2)$$

$A_i$= polynomial regression coefficients
$C_{tr}$= TDS cincentration in reuse, mg/l
$X_i$= flows in Figure 14 p. 64 of Chapter 5, mgd
$Z_4$= capacity of raw water treatment plants, mgd

APPENDIX B

# WATER MANAGEMENT IN THE DENVER AREA

## Introduction

A study of water management in an arid urban area has ramifications which extend beyond the metropolitan boundaries. Denver, Colorado, is a good example of a local water problem of state-wide concern. Water supplies for the city are obtained from sources in both the headwaters of the South Platte River Basin, and the headwaters of the Colorado River Basin. Although water management in the recent past dealt mainly with supply and development, the present and future emphasis can be expected to include water quality control and regional water use efficiency.

Denver evolved from a stopping place for Indians, fur trappers, traders, and explorers prior to 1858 to an expansive metropolitan area containing over half of the state's population in 1973. The catalyst for the founding of Denver was the "Pikes Peak or bust!" gold rush of 1859 stemming from Green Russell's discovery of gold at the confluence of Cherry Creek and the South Platte River in 1858 (Schierbrock, 1960). The initial settlements of Placer Camp and Montana gave way to Auraria and St. Charles, then Auraria and Denver, and finally Denver, the capital of the Colorado Territory and later the State of Colorado. Because St. Charles was actually in the western reaches of the Kansas Territory when the name change occurred, the name selection was made in recognition of the current governor of the territory, James W. Denver. From these early beginnings to the present, Denver's life-blood has been the commerce supported by its water resources.

It is interesting and important to view an area's present conditions in light of the historical events leading to the current status. Much of the social influence responsible for an area's operation can be traced to those times when significant decisions were made and the populace concurred. The structure for administrating an area can often be linked to the regulatory system which evolved as a result of correcting the periodic difficulties experienced in a region. In addition, future events are often best evaluated on the basis of past experience.

This chapter is presented to describe the conditions in the Denver area, especially with regards to water resources and their associated water quality characteristics.

## Regional Characteristics

### Location

The Denver area is located at the eastern base of the Rocky Mountains in the state of Colorado. To the east are the flat high plains and broad rolling prairies, while the regions to the west are mainly mountainous with arid or desert-like valleys. These topographical characteristics have a profound influence on both water quantity and quality. Because the general air flow is west to east, Colorado's water resources are found more abundantly on the western upslope regions than on the eastern side of the mountains. Conversely, most of the state's population is centered along the eastern base. Consequently, water management in Colorado is largely a problem of adjusting the spatial distribution of water resources to satisfy the needs of the people.

Due to its high elevation, Colorado contains the headwaters of four major river basins; (1) Colorado, (2) Rio Grande, (3) Arkansas, and (4) Missouri, as shown in Figure 34. Since these river systems transport water from the state into adjoining states, Colorado has first use of its water resources, a condition which is very advantageous to the water users from a water quality standpoint.

The South Platte River, which passes through Denver as shown in Figure 35, begins in the front ranges of the Rocky Mountains and flows in a northeast direction for approximately 442 miles until its confluence with the North Platte River in Nebraska. Demands for the annual flows generally exceed the available supplies thereby necessitating careful management of the resource.

## Climate

The climatalogic conditions in the South Platte River Basin are primarily a function of elevation, which

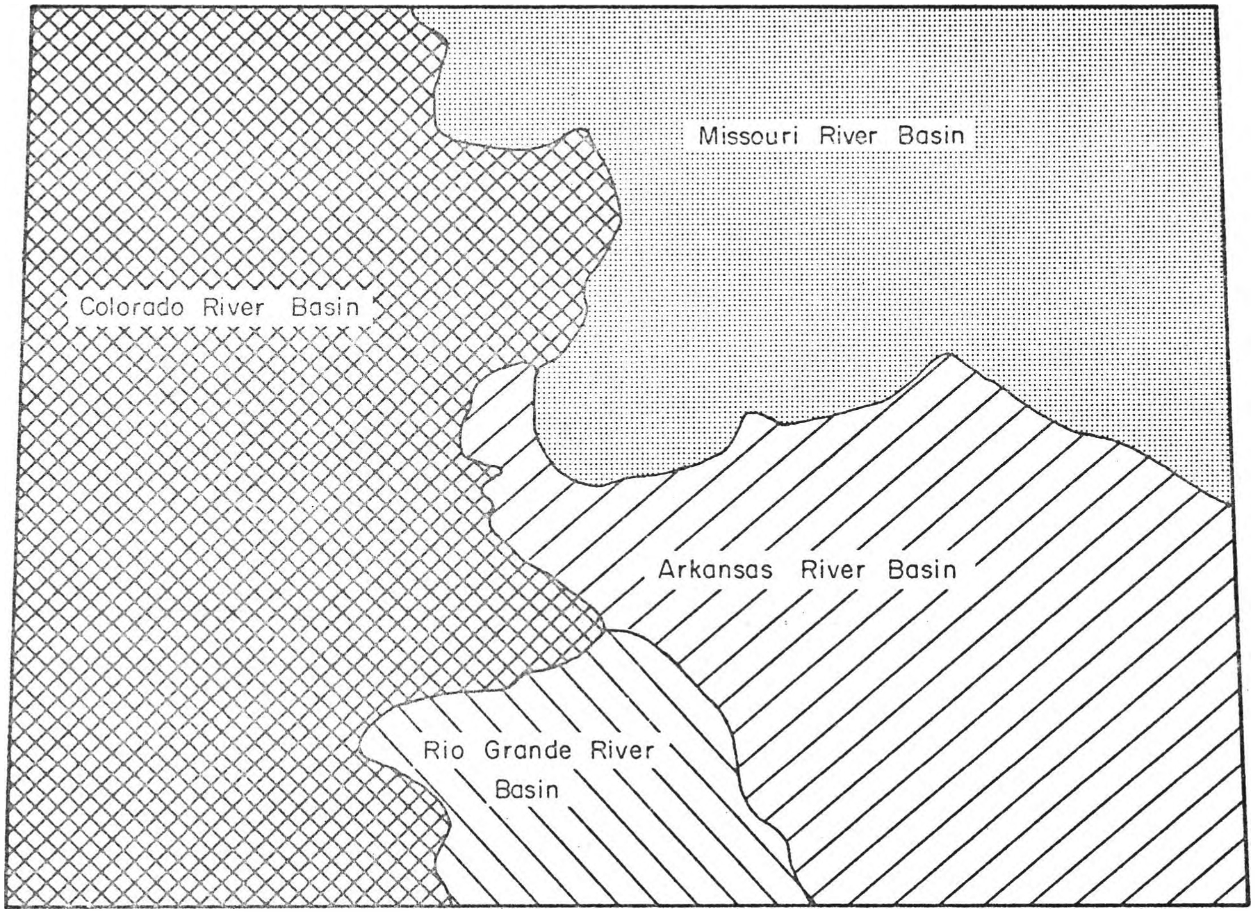


Figure 34. Colorado's major river systems.

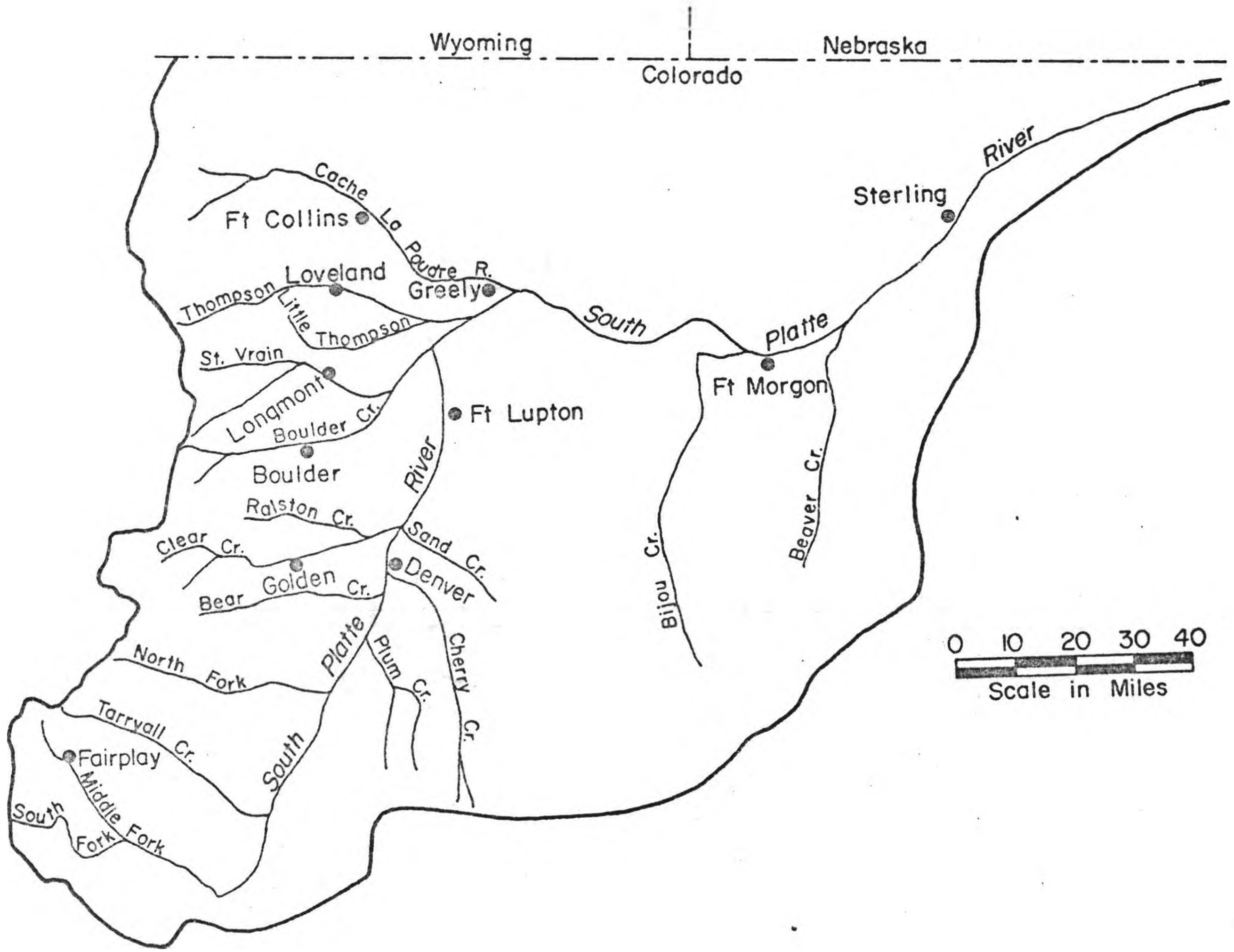


Figure 35. The South Platte River BRsin in Colorado.

ranges from 3,500 feet above sea level in the eastern portion of the basin, to 5,280 feet at Denver, to 14,000 feet in the upper reaches of the watershed. The foothills due west of Denver experience elevational differences of 5,000 feet to 8,000 feet and provide the climatological transistion between the dry, warmer plains and the wetter, colder mountains. The climate in the Denver area, although marked by wide seasonal variations, is characterized by low relative humidity, 12-14 inches of rainfall, and moderate temperatures in both summer and winter.

## Population

The population of Colorado showed an increase of 25.8% between the census of 1960 and 1970, resulting in a current total of about 2.3 million people. Of this total, approximately 74% live in the foothills area between Fort Collins and Pueblo. The Denver metropolitan area accounts for more than one-half of the state's population as illustrated by the historical and projected population trends shown in Figure 4. Much of these increases are due to the net influx of people into Colorado.

## Economy

Colorado's economy has historically been based on its natural resources like mining, agriculture, and recreation. However, the rapid expansion of the states's urban centers lured a large number of diverse industries and supporting activities into the area. Consequently, the present

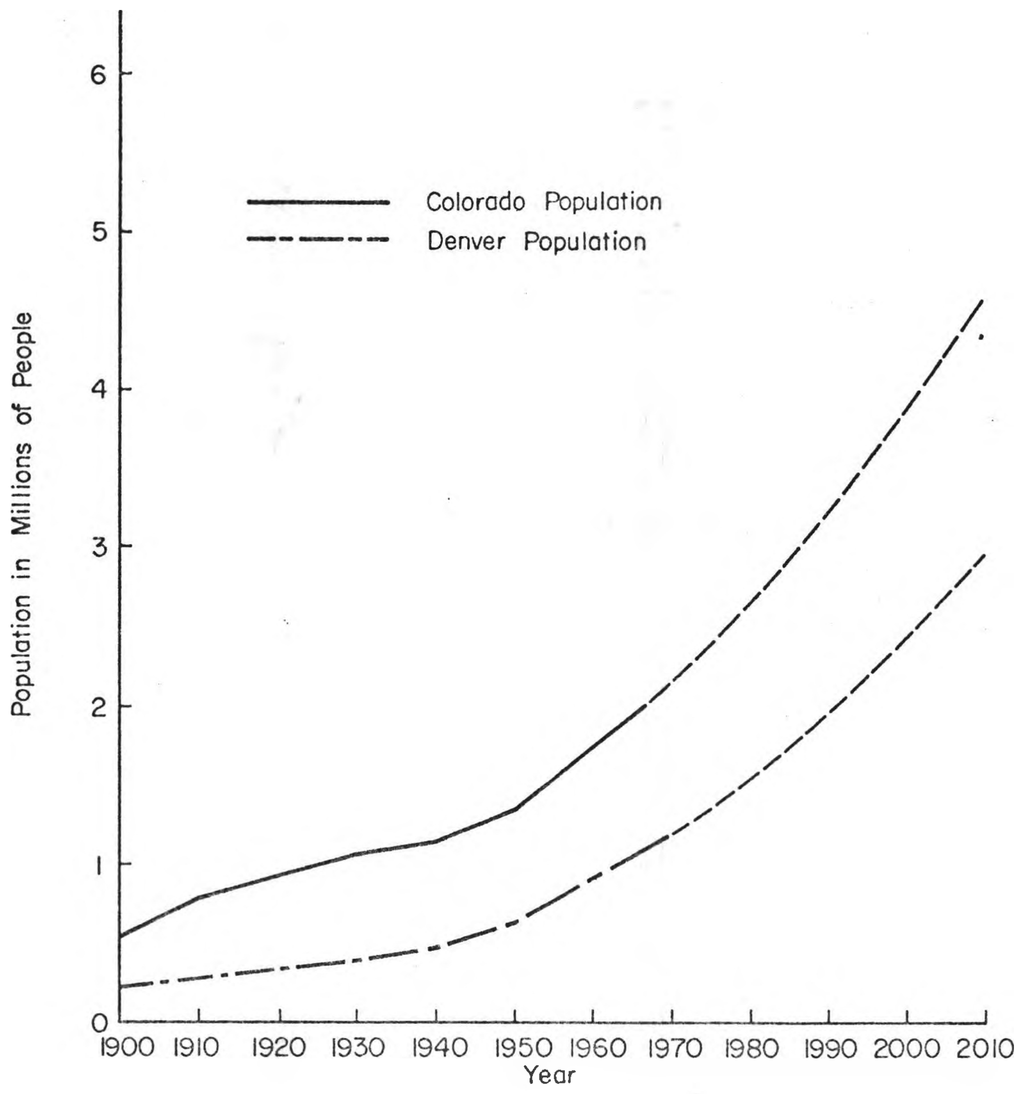


Figure 36. Population trends in the Denver, Colorado area.

economic conditions of Colorado appear to be a well balanced mixture of economic enterprises.

Surrounding the Denver area in the South Platte River Basin, irrigated agriculture constitutes the largest water use. Of the approximately 69% of the basin comprising the agricultural industry, nearly all of it is fertile enough to support profitable alfalfa, small grains, corn, and sugar beet production. In addition, this vigorous agriculture supports related industries including livestock feeding, meat packing, sugar beet processing, milk production, and canneries.

## Historical Water Development and Management

At the time the United States acquired the South Platte River Basin as a part of the famous Louisiana Purchase, four principal overland routes had been established into the Denver area. These trails were along the South Platte, Arkansas, Smoky Hill, and Republican Rivers (Schierbrock, 1960). It was along the South Platte route that the Stephen H. Long expedition of 1819-1820 made an exploratory trip into the area. The journals from this exploration were the first written description of the countryside and received wide readership in the east. However, the report of Major Long was negative in nature, stressing the forbidding nature of the plains to the east and the rugged mountains to the west. Among the conclusions drawn was the inability of the area to support more than a sparce population, nomadic in nature, since the area was unfit for cultivation. Like so many others at the early stages of exploration, Long attempted to access the capability of the land upon the eastern concept of agriculture and, consequently, made an erroneous judgement.

Early water use in the Denver area was agricultural, as would be expected. Residents diverted water from rivers and wells to supplement the croplands with water in the semi-arid environment. In 1859, James McBrown staked a claim on the lower reaches of Bear Creek and with the enactment of Colorado Water Laws, this right became the first priority in the South Platte Basin (Denver Water Department, 1969).

With the practices of constructing ditches to connect and irrigate the lands bordering the stream systems, it became necessary to formulate or devise a legal structure for distributing and administering the water resources. In 1861, the Colorado Territorial Legislature passed a bill which allowed individuals with off-stream land to secure a right-of-way for water crossing adjacent lands and use the water for a beneficial use on these lands. This attitude was a radical departure from the riparian rights doctrine inherited by the eastern areas of the United States from English Common Law. This principle, which eventually became the prior appropriation doctrine, was again alluded to when the territorial Supreme Court decided the case of Unker vs. Nichol in 1872 (Crawford, 1957). In 1876, when the state framed its constitution, the principle of prior appropriation philosophy was included as the state's law. This doctrine states simply that the appropriator who was the first to apply water to a beneficial use also acquires the first right to that water.

Because most of the early appropriations were for agricultural use, the water available for urban areas were largely based on junior rights. However, as the urban areas grew, the water formally used for irrigation was converted for municipal uses by a transfer of the water rights from one use to another. Such transfers is the manner in which most domestic supplies have developed (Denver Water Department, 1969). Although other projects had been conceived, the first successful attempt to bring domestic water to the residents was made by the Denver City Water Company in 1872.

By the early 1900's, all the dependable flow of the South Platte River and its tributaries had been appropriated for use, principally as supplemental irrigation water. Although the application of water to the farm lands had greatly stabilized the base flow, flood flows were common and could not be utilized. Due partly to these flood losses, and the junior nature of Denver water rights, the Denver Union Water Company, organized in 1894, built Cheesman Dam and reservoir in 1905 to collect these surplus flows.

In 1918, the Denver Board of Water Commissioners assumed control of Denver's water supply system which had as its major source of water the surface water of the South Platte River. Around this time, however, it became evident that within planning horizons the water rights for the South Platte's water would soon be completely utilized. As a result, planning for alternative supplies such as interbasin diversions was begun.

Development of the South Platte River as a source of water supply for the Denver area essentially ended in 1932 with the completion of the Eleven Mile Canyon. Up until this time, Denver had either built or purchased Marston Lake, Cheesman Reservoir, and Antero Reservoir, the major reservoirs on the South Platte system. Raw water is stored in these reservoirs and then brought down the South Platte to Denver's raw water treatment system through a series of regulatory reservoirs.

With this maximum development of the South Platte water, Denver was in a good position to justify the diversion of water from the western slope. The early planning performed by Denver proved very valuable in obtaining western slope water rights needed for diversion projects to be successful. The first trans-mountain diversions to serve as additional supplies to Denver's water supply system came with the completion of the Fraser system in 1936. This water flows through the six-mile-long Moffat Water Tunnel after being collected from the Fraser River and its tributaries on the western slope. Development of this water tunnel was tied in very closely with the development of the Moffat railroad tunnel. In fact, Moffat Tunnel is the pioneer bore of the railroad tunnel.

In 1955, the Board of Water Commissioners acquired the Williams Fork Collection System and the three-mile-long A. P. Gumlick Tunnel (formerly Jones Pass Tunnel). This system had been constructed in the 1930's by a grant from the Public Works Administration to the Denver Public Works Department. The Williams Fork system was connected to the Fraser system in 1958 through construction of the Vasquez Tunnel. Consequently, water from the Williams Fork system now goes to the eastern slope via the Gumlick Tunnel, but rather than go down Clear Creek to Denver, the water travels back to the western slope via the Vasquez Tunnel and enters the Moffat Tunnel. This is accomplished so that water from the Williams Fork system can be stored, along with the Fraser system water, in Ralston Reservoir constructed in 1937 and Gross Reservoir completed in 1955. Prior to completion of the Blue River diversion project, the Fraser-Williams Fork system supplied almost 50 percent of Denver's municipal water supply (Board of Water Commissioners, 1971).

The largest diversion project to be completed by the Board of Water Commissioners is the Blue River Diversion System. Initial work on this system can be traced back to studies performed in the early 1920's, but was delayed until about 1955 by legal entanglements. With the passage of a $75 million bond issue in 1955, and a supplement of $40 million in 1959, construction was begun in earnest. The key part of the system, the Harold D. Roberts Tunnel, was completed in 1962, is 23.3 miles long and has a dog-leg to the south. Its western portal is located at Dillon Reservoir, elevation 8,844 feet, while the eastern portal is at Grant, Colorado, on the North Fork of the South Platte River, 174 feet lower than the west portal.

The major storage facility in this system, Dillon Reservoir, was completed in 1963 with an effective storage capacity of 254,000 acre feet.

The Denver Board of Water Commissioners in continuing to plan for future demands, submitted a $200 million bond issue to the people of Denver. But, in refusal of past support, they turned down the bond in 1972, which would have permitted the development of the Eagle-Piney Collection System. This system would have added an additional 100,000 acre-feet of water to the Denver municipal water system. Through an intricate system of tunnels, canals and reservoirs, it would have transported Eagle-Piney water to Dillon Reservoir for transmission to Denver via the Roberts Tunnel (Board of Water Commissioners, 1971).

The diversion of water from the western slope to the eastern, has not been accomplished without a lengthy and costly battle over water rights. There have been, and continue to be, controversies of water rights and it is doubtful that any type of agreement will end the controversy indefinitely.

## Present Conditions

Although Denver is located in the rain shadow of the eastern slopes of the Rocky Mountains, investment of time, money, and technology have been successful in redistributing water resources to supply local demands. Those responsible for acquiring, treating, and delivering water supplies must insure a dependable supply even in long periods of drought. To date, the Denver area water planners have been relatively successful in accomplishing this objective by comprehensive and long range analysis of needs and trends. In the recent past, the growth and merger of the city of Denver and the communities in the surrounding counties prompted study on a metropolitan basis. Consequently, this study will also include these dimensions to the extent that the Denver water and wastewater facilities connect with the others.

Periodically, it is interesting to examine existing conditions in order to better evaluate the needs for future decisions. These existing conditions are also well defined and readily available so the effects of future decisions can be extrapolated from existing information.

Available Water Supplies

From the two major sources, the South Platte and Colorado River Basins, Denver has currently a usable water supply of about 310,000 acre-feet annually. The distribution of this supply is composed of about 61,000 acre-feet from the Moffat System, 168,000 acre-feet from the Blue System, and 81,000 acre-feet from the South Platte rights (Hobbs, 1971).

The availability of the flows which serve these systems is not continuously congruent with the demand distribution, so storage and distribution reservoirs have been constructed and maintained for adjusting local hydrology to the pattern of the needs. In the South Platte system, the storage capacity of Lake Cheesman, Eleven Mile, Antero, and a portion of Soda Lakes reservoirs amounts to over 193,000 acre-feet. This along with the 43,000 acre-foot Gross Reservoir in the Moffat System and 254,000 acre-feet in Dillon Reservoir of the Blue System provide Denver with a storage capacity of about 490,000 acre-feet (Board of Water Commissioners, 1972). In order to adequately supply the wide variations in monthly and daily demands, operation reservoirs serving the system have been implemented to yield a capacity of over 30,000 acre-feet. These reservoirs include Platte Canyon, Long Lakes, Ralston, and Marston Lake. In order to pictorially view these reservoirs, Figure 37 has been included.

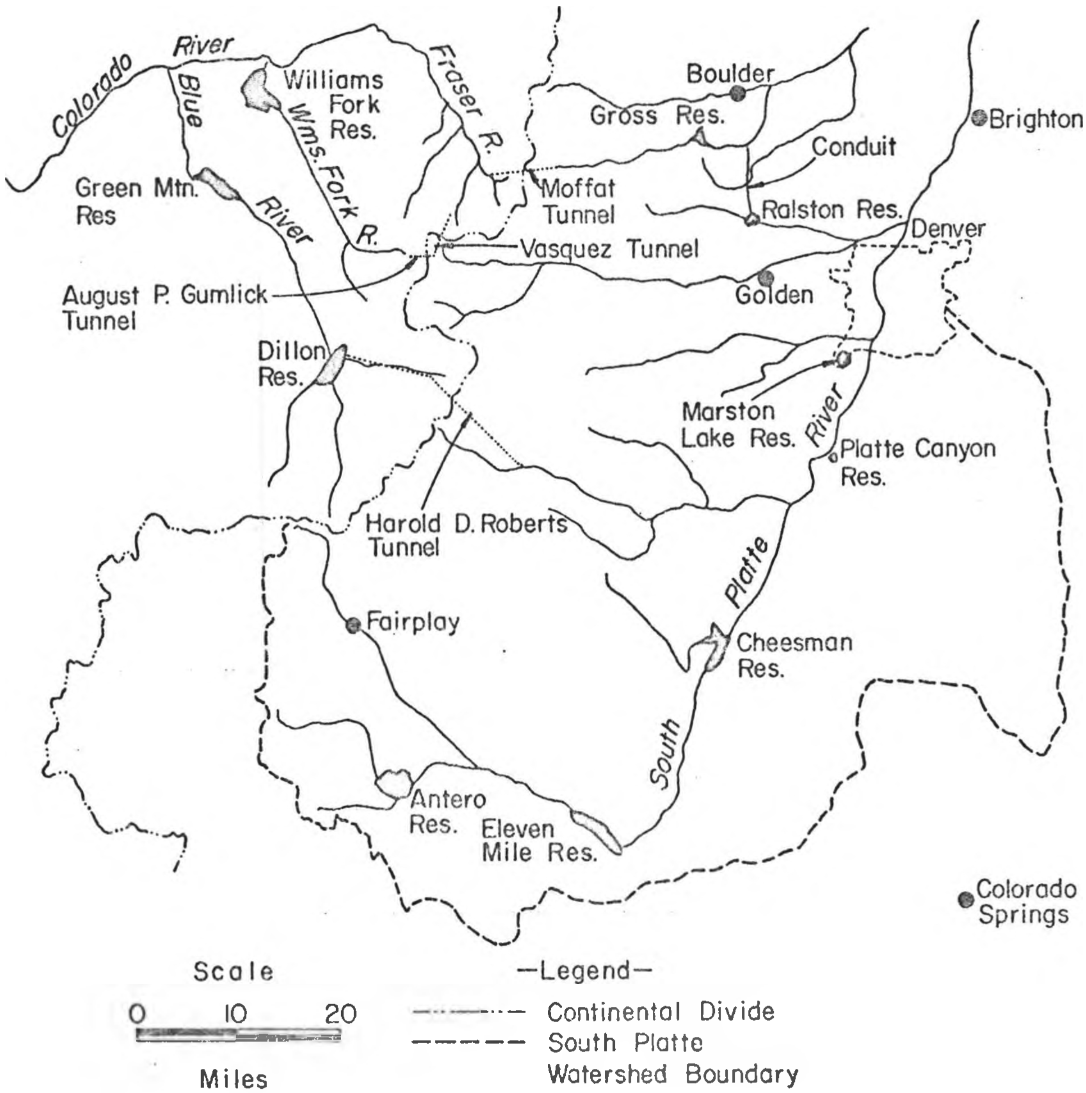


Figure 37. Denver's water system storage and operating reservoirs ( Board of Water Commissioners, 1972).

The water quality of the flows supplied to Denver users is far below the upper limits placed on domestic, municipal, and industrial waters. A continual monitoring program is undertaken by the Board of Water Commissioners, U.S. Geological Survey, and Colorado Public Health Department as required by the purposes of these organizations. Although all important water quality parameters are checked, of interest in this study are the BOD and TDS concentrations. The Board of Water Commissioners (1971) lists water quality characteristics of the South Platte supplies. TDS levels in these flows average about 150 mg/l, but have reached highs as much as 300 mg/l. Iorns, Hembree and Oakland (1965), in an exhaustive study of Upper Colorado River Basin water resources, show TDS levels in the upper reaches of the watershed to be about 100 mg/l. This figure is also verified by Denver Water Department analyses. BOD concentrations in the total water supply is insignificant, indicating as well that color, turbidity, and fecal coliforms are minimal.

Raw water treatment is an absolute necessity even though the water supplies are of high quality. The first treatment facility, the Kassler Plant, was built in 1890 to process water from the South Platte River with underground filtration galleries. Then in 1906, the plant was enlarged to its present capacity of 50 mgd and converted to the slow sand filtration process. Then in 1925, the North Side Marston Treatment Plant was constructed which added an additional 100 mgd to the existing system. Along with this duo-media rapid sand filtration plant, a 60 mgd addition was added in 1961 and another 100 mgd addition in 1967 was added to treat western slope water. The remaining treatment plant, the Moffat Water Treatment Plant was completed in 1937 to treat Moffat Tunnel imports. This treatment plant, which originally had a capacity of 80 mgd, was expanded in 1957 to 150 mgd. Together, these raw water treatment plants give Denver a 460 mgd capacity (Board of Water Commissioners, 1972). The location of these water treatment plants is shown in Figure 38.

## Demands

To characterize the demands of a large municipal area such as Denver, several factors should be examined. For example, the time varying aspects of the demands are important planning and design parameters. In addition, the distinctive nature of the demands presented in the previous chapter suggest that water quality requirements and consumptive use characteristics are variables needing evaluation in order to make a more effective use of the water.

The Board of Water Commissioners (1971) present monthly water demands based on both a ten-year average and for the 1971 year. These data have been included in Figure 39. It is interesting to note the large increases during the peak use months of the summer, which indicate the use of water for irrigation of lawns, trees, and shrubs. If

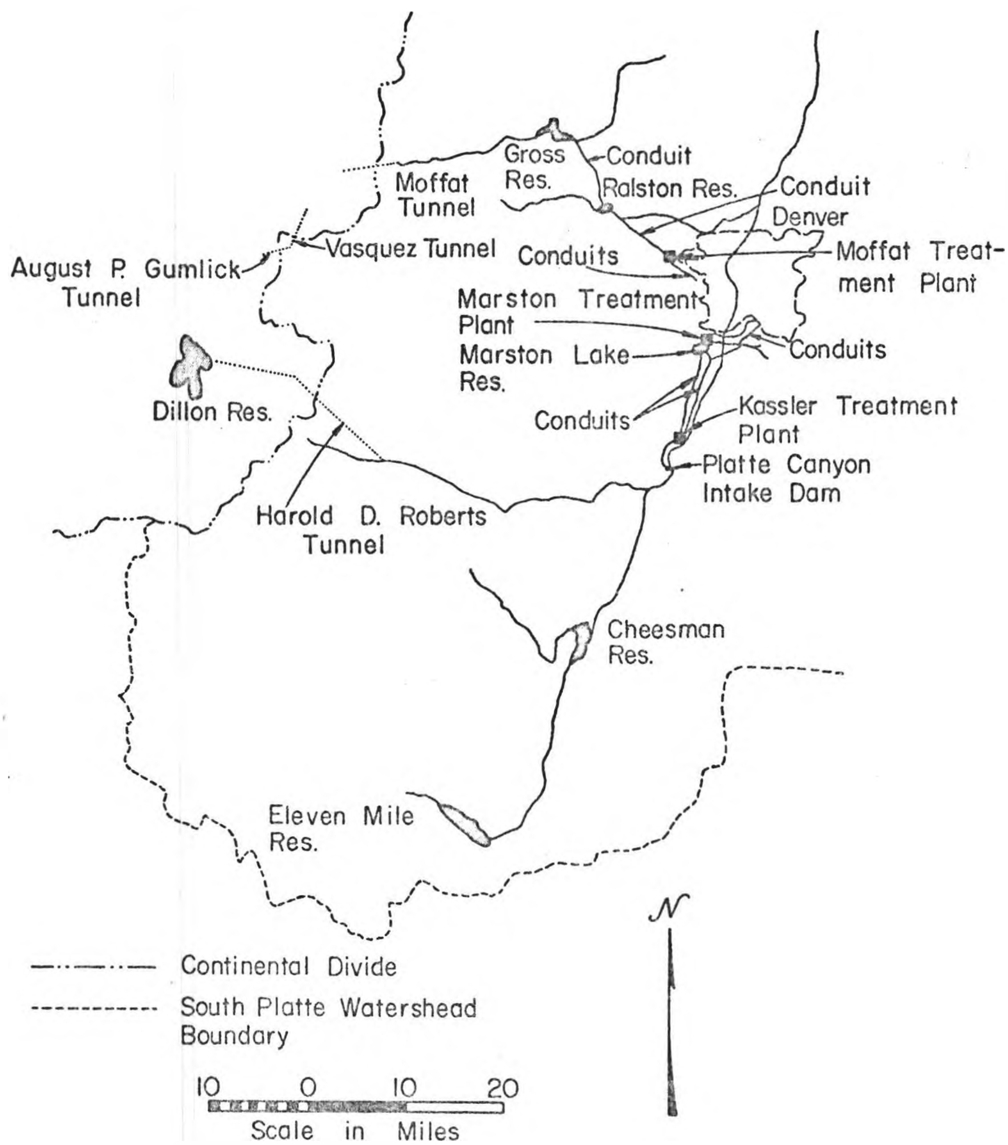


Figure 38. Existing raw water treatment facilities for the Denver supply network ( Denver Water Department,1969).

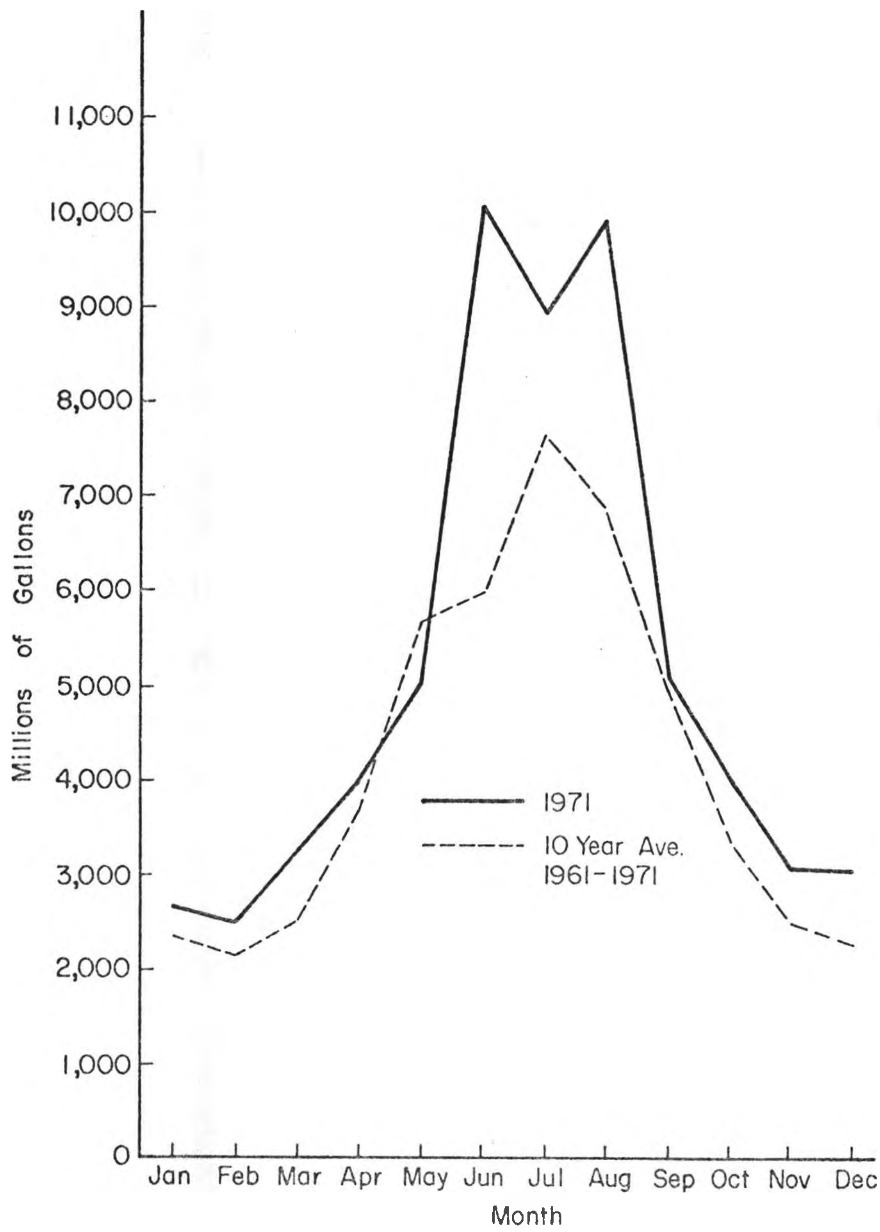


Figure 39. Monthly distribution of Denver water demands (Board of Water Commissioners, 1971).

the maximum day is divided by the average daily demands, the excess capacity factors for design of raw water treatment can be determined. This ratio during the 1970 water year was approximately 2.6. According to the Denver Water Board (1969), nearly 40% of the urban water supply was used for the municipal type demand, which verifies the cause of peaking in the hot summer months.

Although the actual per capita water use in the Denver area is close to 60 gallons per day, the total consumption divided by the population shows a steadily increasing rate. During 1971 it was on the order of 200 gpd. The reasons for these high consumption rates are explained by Denver water planners as increased industrial activity, expanding area, and a more affluent population.

## Wastewater Collection and Treatment

The collection and treatment of wastewater in the Denver metropolitan area is presently unable to achieve the level of water quality control set forth by state and federal regulatory agencies (U.S. Environmental Protection Agency, 1972). The present system, shown in Figure 40, consists of fifteen major treatment systems serving more than 800,000 people and numerous industrial enterprises. Of this system, nearly 85% is served by the combined facilities of the North Denver Wastewater Treatment Plant (primary only), and the Metropolitan Denver Sewage Disposal

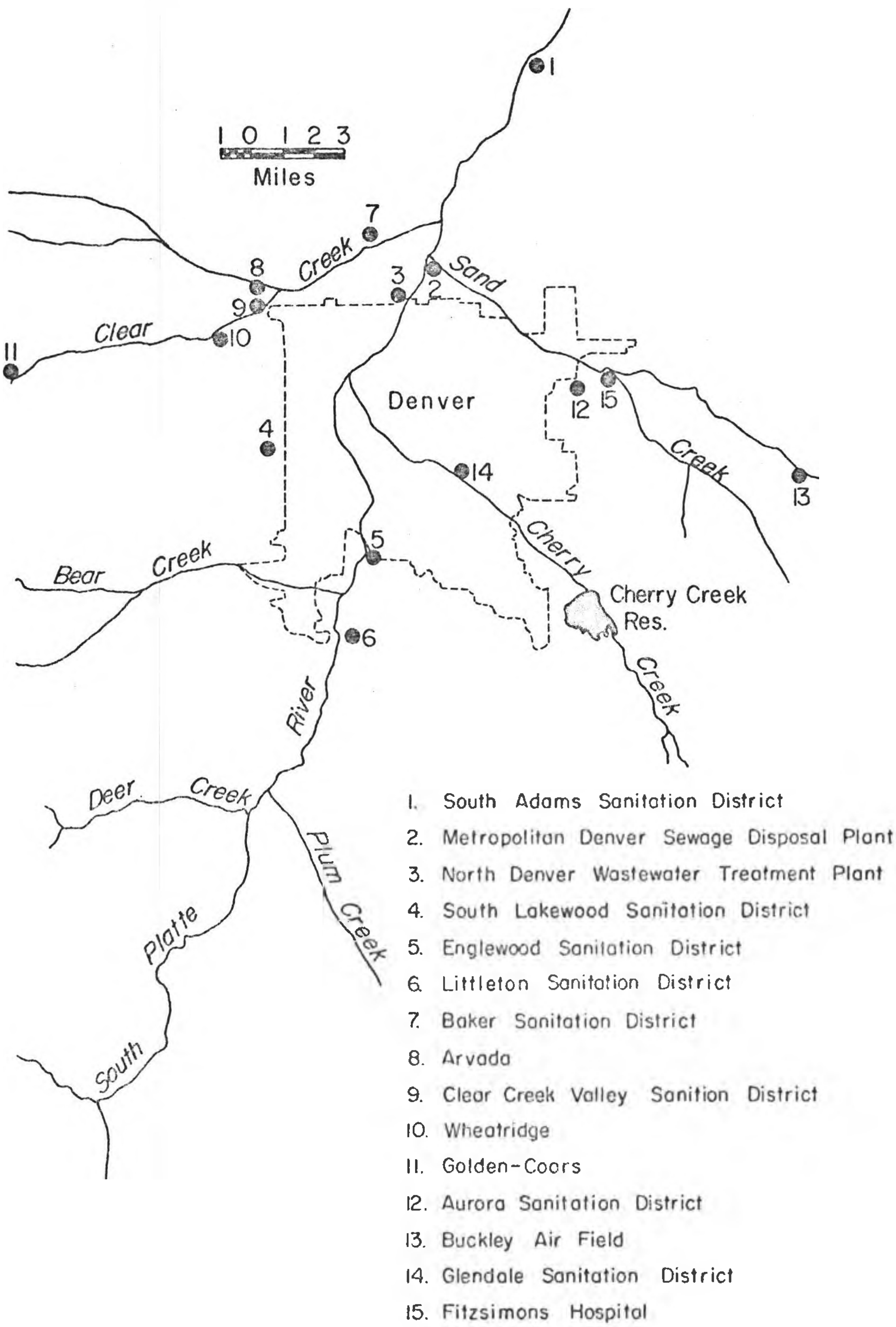


Figure 40. Municipal wastewater treatment facilities in the Denver area ( U.S. Environmental Protection Agency, 1972).

District Plant #1. Consequently, the description presented herein can be limited to the flows at these two locations.

Prior to the completion of the metro plant, the majority of the flows were subjected to only primary treatment. As a result, the South Platte River in the Denver area was severely polluted and steps to alleviate this condition were investigated. During three study periods extending from August of 1964 to October of 1965, this reach of the river system was extensively examined by the South Platte River Basin Project of the Federal Water Pollution Control Administration (U.S. Environmental Protection Agency, 1972). The results indicated that the quality deterioration occurring through the city area in terms of dissolved oxygen ranged from 6-10 mg/l at the 19th Street station to 1-3 mg/l at York Street, to 0.2-4 mg/l at the vicinity of 88th Avenue. BOD concentrations increased from 10-20 mg/l at the 19th Street sampling point to 45-170 mg/l at the downstream locations. In addition, the density of fecal coliform bacteria was extremely high, exceeding one million organisms/100 ml at both York Street and 88th Avenue (Federal Water Pollution Control Administration, 1966).

As a result of these studies, recommendations were made to state and local authorities. The Colorado Water Pollution Control Commission in compliance with Public Law 84-660, Federal Water Pollution Control Act, submitted stream standards and classified the flows in the South Platte River accordingly.

During the August-December period of 1971, the personnel of the Environmental Protection Agency conducted additional water quality investigations in the South Platte River Basin (U.S. Environmental Protection Agency, 1972). These studies not only included the stream surveys as in the previous studies, but also an in-plant survey of the Metropolitan Denver Sewage Disposal District Plant #1, the North Denver Wastewater Treatment Plant, and nine of the satellite plants shown in Figure 40. The purpose of this follow-up investigation was primarily to evaluate the success of the abatement efforts to that date. Some of the important conclusions reached included:

(1) The North Denver Treatment Plant had BOD removal efficiencies ranging from minus 11 percent to 58 percent, but according to plant records, average between 22 and 36 percent. Because the sewage collection system is also a storm water drainage network, high flows of raw sewage are occasionally spilled directly into the river. In addition, the periods of poor removal efficiencies cause difficulties such as overloading in the secondary treatment facilities of the metro plant.

(2) The metro plant, overloaded both hydraulically and organically with peak flows exceeding the design capacity by 60 mgd and the organic loading by 10 percent. Four of the twelve aeration bases are being used for sludge digestion.

(3) Adequate treatment was not being provided by the metro plant for BOD, resulting in an average discharge of about 30,200 lb/day. Including the removal of the North Denver facility, BOD removals for the metro plant ranged from 63 percent to 96 percent on a daily average and were below the state requirement of 80 percent BOD removal 20 percent of the time.

Improvements are continually underway to reduce the contaminants contained in the effluents from this urban area. From the first sanitation district, called the 16th Street Sanitation District in 1882, to the metro concept of the 1970's, wastewater treatment has been among the goals of the Denver area. Current conditions have been defined by Henningson, Durham, and Richardson (1970), and reviewed by Alexander Potter Associates (1970). Data collected by these investigators, as well as reports on plant loadings by the U.S. Environmental Protection Agency (1972) and the Metropolitan Sewage Disposal District #1, are summarized in Table 4 to indicate the presently encountered wastewater conditions in the Denver area.

Table 4. Wastewater characteristics of the Denver metropolitan area.

| | Influent | Effluent |
|---|---|---|
| **North Denver Wastewater Treatment Plant** | | |
| Average Daily Flows, mgd | 85 | - |
| Peak Daily Flows, mgd | 153 | - |
| Average Daily BOD, mg/l | 270 | 180 |
| Average Daily Suspended Solids, mg/l | 260 | 150 |
| Average Daily TDS, mg/l | - | - |
| **Metropolitan Sewage Disposal District #1** | | |
| **Primary Treatment** | | |
| Average Daily Flows, mgd | 22 | - |
| Peak Daily Flows, mgd | 41 | - |
| Average Daily BOD, mg/l | 460 | 350 |
| Average Daily Suspended Solids, mg/l | 420 | 160 |
| Average Daily TDS, mg/l | - | - |
| **Metropolitan Sewage Disposal District #1** | | |
| **Secondary Treatment** | | |
| Average Daily Flows, mgd | 107 | - |
| Peak Daily Flows, mgd | 190 | - |
| Average Daily BOD, mg/l | 214 | 31 |
| Average Daily Suspended Solids, mg/l | 152 | 56 |
| Average Daily TDS, mg/l | 739 | 739 |

## Future Developments

Although the future is completely unknown, it is nevertheless necessary to plan the future delivery of goods and services in order to insure their availability. Since many variables influence the outcome of future events, the planning process is forced to rely on extensions of past experience. Such a basis for prediction has been repeatedly demonstrated as ineffective in dynamic societies, but no better alternative is currently available.

Urban water planners are faced with a dangerous task. Because of the institutional constraints, the time between project conception and water delivery may be as much as 30-50 years for many large projects. In such cases, the designs must be based on 50 year demand projections which are difficult if not impossible to formulate. In addition, the question of whether it is more desirable to emphasize current needs rather than future conditions nearly always arises. A good example is the political and economic philosophy regarding interest or discount rates in project feasibility evaluations. Consequently, the urban water planner is charged with meeting a demand at the end of the planning horizon, at minimum cost (political, social, economic), subject to the restrictions of an immense administrative structure. Some of the expected conditions in the Denver area are discussed below.

The most commonly used tool in projecting the aggregate demand is the per capita consumption. Since the principal variable in the demand function is population, total demands can be shown to be related to the population. The metropolitan area population can be delineated as shown in Figure 41. Based on historical data, per capita consumption can be projected as illustrated in Figure 42.

Another important consideration in facility planning is the hourly, daily, and monthly demand characteristic. System storage allows treatment plant capacities to be designed on maximum day requirements. The nature of the Denver demands are shown in Figure 43. It is interesting to note the widening gap between maximum day and average day demands. This condition places particular emphasis on careful planning since the construction costs for these treatment facilities are high.

## Water Supplies

The future water supplies for the region will be supplied from the present sources. Better storage management and gradual acquisition of in-basin water rights will expand the South Platte supply from the 80,000 - 90,000 acre-feet presently to 110,000 acre-feet in 1990 and 121,000 acre-feet in 2010. Of course, these figures are based on the safe annual yield concept which tends to be conservative (Denver Water Department, 1969).

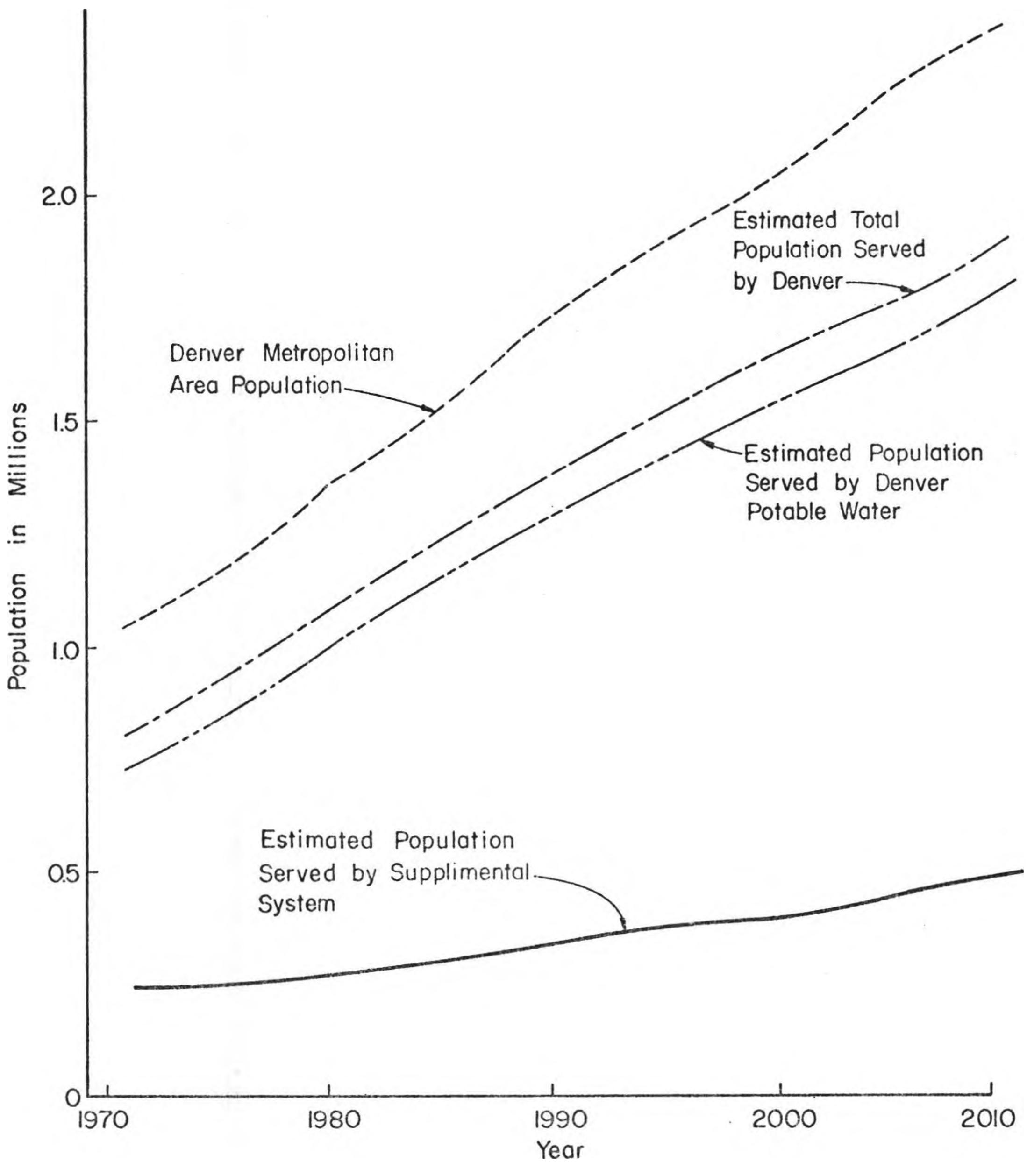


Figure 41. Denver metropolitan area population trends ( Board of Water Commissioners, 1972).

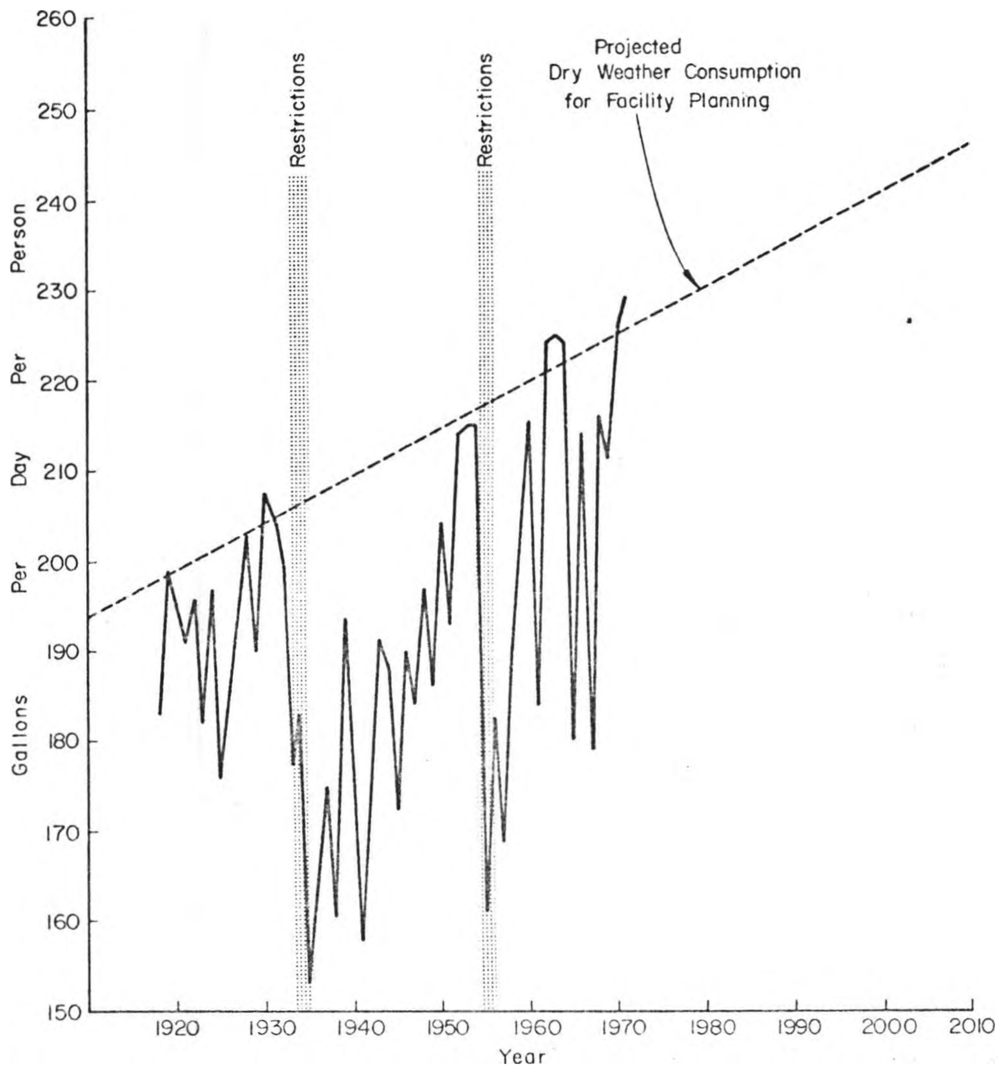


Figure 42. Historical and projected per capita water consumption in Denver ( Board of Water Commissioners, 1972).

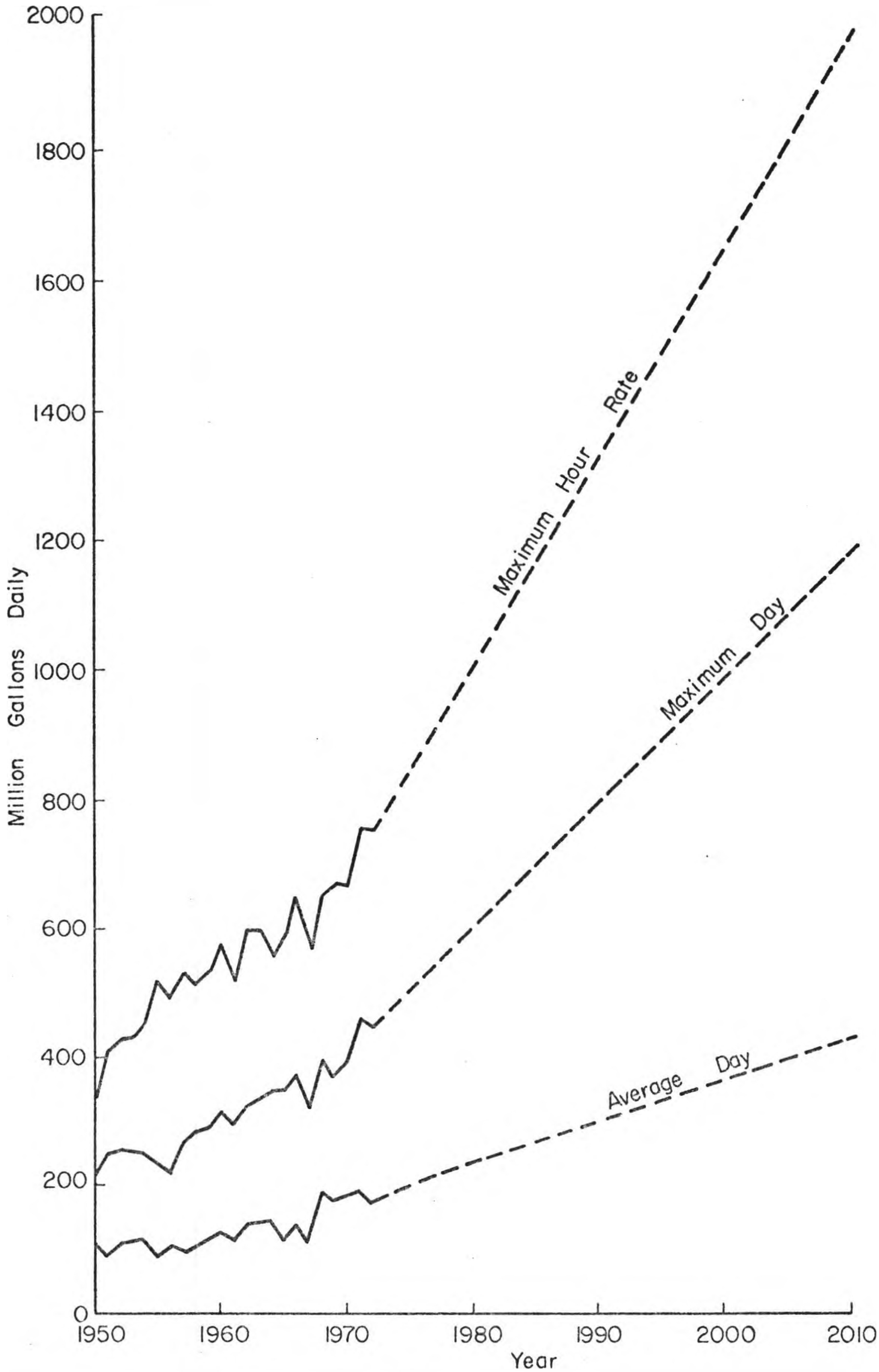


Figure 43. Denver's treated water demand characteristics, historical and future ( Board of Water Commissioners, 1972).

The Moffat system is expected to be increased from present capacity of 70,000 acre-feet to 122,000 acre-feet in 1990 and 136,000 acre-feet in 2010. However, these expansions, along with the doubling of the Blue network, will require large capital outlays for new construction. Supply costs in present-worth form are listed at $750 per acre-foot (Denver Water Department, 1969).

## Water Quality Management

There seems to be little doubt that more stringent water quality controls will be required in the immediate future. Recent Federal legislation has adopted the tentative philosophy of zero pollutant discharge, but the ability of regulatory agencies to accomplish such comprehensive controls remains to be seen. State pollution control agencies are also formulating schedules for increasingly rigid effluent standards.

Except for agricultural return flows, water quality management has been primarily concerned with organic pollutants. However, the most serious water contaminant may very well be the concentrations of dissolved solids, or salinity. All water uses in which water is consumptively used concentrate these salts, but some uses such as agriculture and urban uses add additional salts to the system. As a result, TDS standards may be expected to be imposed in the near future as well.